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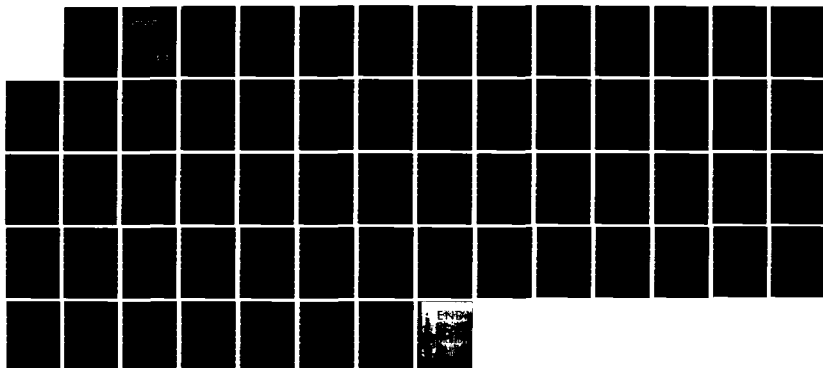
THE ANALYSIS OF CRACK GROWTH DATA WITH APPLICATION TO  
TRIP STEEL(U) ARMY MATERIALS AND MECHANICS RESEARCH  
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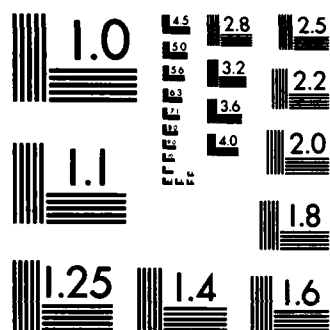
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# THE ANALYSIS OF CRACK GROWTH DATA WITH APPLICATION TO TRIP STEEL

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MECHANICS AND STRUCTURAL INTEGRITY LABORATORY

March 1983

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ABSTRACT

A procedure is advocated to replace the common practice of establishing a rate of crack growth law directly from increments of growth or from slopes of smooth curves drawn through crack length vs number of loading cycles data. In the advocated procedure, a postulated rate equation such as, for example, the power law  $da/dN \sim (\Delta K)^m$ , is integrated for the specimen geometry used and values of the integral corresponding to experimental crack lengths are plotted against cycles of loading to reach these lengths. Thus, where, and if, the rate equation is valid the plot will consist of one or more straight line segments for each of which constant parameters may be obtained, or confirmed if theoretical. Intersections of straight line segments will correspond to crack rate of growth discontinuities unobservable by the usual method. Using Trip Steel data supplied by Syracuse University as an example, it was found that all growth can be proportional to  $(\Delta K)^2$  with a proportionality constant that changes discontinuously at various amounts of growth in general conformance to the author's experience and his crack growth law. This conclusion is not invalidated by a gradual decrease in loading during testing, or by the differences in analytical expressions for  $\Delta K$  found by different investigators. It is shown that, although a rate proportional to  $(\Delta K)^4$  is not that for any segment of the whole growth curve, it represents the envelope to such segments and thus is a sort of overall representation that may be useful in design. The discrepancy between a rate proportional to  $(\Delta K)^m$ ,  $m = 3.7$ , found by Syracuse University and the rate proportional to  $(\Delta K)^2$  is explained by showing that the Syracuse method of analysis gives a power corresponding to an envelope.

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*A*

## NOMENCLATURE

CTS	Abbreviation meaning compact tension specimen of ASTM Standard E399-70T
N	Cycles of loading
B	Thickness of CTS (Figure 1)
w	Width of CTS (Figure 1)
a	Crack length measure for CTS in ASTM Standard E399-70T formula for K (distance from loading axis to crack front)
ICL	Initial (machined) crack length of CTS (Figure 1)
GCL	Grown crack length of CTS ( $a - ICL$ ) (Figure 1)
x	$x = a/w$ in ASTM CTS formula for K [Equation (7)]
K	Stress intensity factor
P	Load on CTS
$\Delta P$	$P_{\max} - P_{\min}$ in CTS loading cycle
$\Delta K$	$K_{\max} - K_{\min}$ in CTS loading cycle
$\Delta P_{\text{mean}}$	$(\Delta P, \text{ start of test} + \Delta P, \text{ end of test})/2$
$\Delta K_{\text{mean}}$	$(\Delta K, \text{ start of test} + \Delta K, \text{ end of test})/2$
$I_{gm}$	Integral for growth rate $\sim (\Delta K)^m$ for geometry g, e.g., $g \equiv cc$ for a center cracked specimen
$I_{CTSm}$	CTS integral for growth rate $\sim (\Delta K)^m$
$I_{CTSmD}$	$I_{CTSm}$ modified for decrease (D) in load during test
$f(a/w)$	From formula for $\Delta K$ , i.e., $\Delta K = (\Delta P/B\sqrt{w})f(a/w)$

## INTRODUCTION

The test results analyzed here were supplied by D. N. Lal and V. Weiss of Syracuse University, who obtained them to satisfy Navy-Syracuse Contract N00140-70C-0126.

This contract was concerned with the mechanical properties of Trip Steels, including toughness and crack growth characteristics.

In this connection Lal and Weiss made an experimental study<sup>\*</sup> of crack growth under repeated loading using an ASTM Compact Tension Specimen (CTS). They analyzed the growth with unusual care in that they made all their comparisons of the rate of crack growth for a specific crack length of 0.6 in. from the line-of-action of the loading pins, different specimens being used for each different loading, i.e., different stress intensity parameter  $\Delta K$ . On each test the minimum load used was approximately one-tenth of the maximum load.

Their main conclusions pertinent to the present study, were that Trip Steel offered no special advantage in cyclic crack growth resistance and that the rate of crack growth was almost proportional to the three and seven-tenths (3.7) power of the cyclic range of toughness parameter  $\Delta K$ .

Not only was AMMRC interested in the potential of Trip Steels in general, but in formulae for the toughness parameter  $K$  and the crack growth characteristics in particular. Beeuwkes of AMMRC had personally and indirectly analyzed experimental crack growth curves of a large number of materials in accordance with his crack growth formula and invariably found that the growth could be very closely matched by making the growth rate proportional to the second power of  $\Delta K$  in constant stress amplitude tests. Thus, it was of interest to both Syracuse University and AMMRC to determine whether or not Trip Steels behaved differently in crack growth caused by repeated loading, than the other materials analyzed and, in particular, whether the rate of crack growth conformed to the Beeuwkes formula which, essentially, makes yield strength the only adjustable rate-controlling material property. Accordingly, Syracuse University turned over to AMMRC all its crack growth data on this steel for further analysis. This was carried out as described in this report, and as will be seen, required the production of a number of formulas and tables, as well as a comparison of toughness formulae for  $K$  proposed by different sources, all of which may be useful in other investigations of crack growth.

It is found that the results obtained often strongly depended upon the method of analysis of the experimental data. Comparisons are made with the procedures advocated and applied here to all the test results. Accordingly, this report may be considered to be a primer of what are here regarded as basic

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\*The Syracuse findings were published in WEISS, V., LAL, D.N., and BLOCK, U. Mechanical Behavior of Trip Steel as a Function of Fabrication and Processing Variables, Dept. of Chem. Engr. & Matls. Sci., Syracuse University, New York, MET-VW-1826-0272FR, February 1972, Submitted to Naval Ship and Development Lab., Annapolis, Md. 21402, Code A933, received after the present analysis was carried out.



concepts of a proper analysis procedure. Tables to facilitate use of such a procedure are provided.

## SYRACUSE UNIVERSITY TESTS AND METHOD OF ANALYSIS

The Syracuse University work covered the mechanical behavior of Trip Steels as a function of fabrication and processing variables, Table 1.

TABLE 1. FABRICATION AND PROCESSING OF STEELS TESTED BY LAL AND WEISS

MATERIAL DESIGNATION	ALLOY COMPOSITION	CONDITION	INITIAL PROCESSING	TRIP PROCESSING
Heat 2321 Type I (B <sub>1</sub> )	C-0.27, Mn-0.90, Si-1.88, Cr-8.80, Ni-8.5, Mo-4.0	Final Thickness 0.300 in.	Plate hammer forged 1180C(2150F), Austenitized 1204C (2200F), Cool to room temperature	80% warm roll 426C(800F), Temper 350C (660F) 1 hr.
<u>Yield Strength: 220,000 psi longitudinal; 200,000 psi transverse</u>				
Heat 2322 Type I (B <sub>2</sub> )	C-0.27, Mn-0.91 Si-1.84, Cr-8.81, Ni-8.73, Mo-4.07	Final Thickness 0.285 in.	Plate hammer forged 1180C(2150F), Austenitized 1204C (2200F), Cool to room temperature	80% warm roll 426C(800F), Temper 350C (660F) 1 hr.
<u>Yield Strength: 208,000 psi longitudinal; 206,000 psi transverse</u>				
Heat 2322 Type II (B <sub>3</sub> )	C-0.27, Mn-0.91, Si-1.84, Cr-8.81, Ni-8.73, Mo-4.07	Final Thickness 0.315 in.	Plate hammer forged 1180C(2150F), Austenitized 1204C (2200F), Cool to room temperature	80% warm roll 426C(800F) 15% Cold work, Temper 350C(660F), 2 hrs.
<u>Yield Strength: 220,000 psi longitudinal; 208,000 psi transverse</u>				

Note: In the present report, Young's Modulus of Elasticity was assumed to be  $E=28 \times 10^6$  psi.

The nominal composition was 0.27 C, 0.90 Mn, 1.86 Si, 8.80 Cr, 8.7 Ni, and 4.0 Mo. Two heats were employed and two Trip Steel processes were used on one of the heats giving a total of three materials:

1. B<sub>1</sub> (Heat 2321, Type I)
2. B<sub>2</sub> (Heat 2322, Type I)
3. B<sub>3</sub> (Heat 2322, Type II).

All three materials were tested for crack growth characteristics. As a result, it was concluded in the Syracuse University work that a fatigue crack propagated through the material according to the rather generally-used power law rate of crack growth per cycle relationship,

$$\frac{da}{dN} = C_m (\Delta K)^m \quad (1)$$

with  $m \cong 3.7$ . Here  $\Delta K$  is the maximum value of  $K$  minus the minimum value of  $K$  in the loading cycle. (The Beeuwkes relationship that we shall employ in part has  $m = 2$  and a theoretical value for  $C$ .)

Lal and Weiss' test method consisted of fatiguing about four ASTM-types of CTS (Figure 1) of each material at different stress intensity amplitudes  $\Delta K$  with  $R = \text{minimum load/maximum load} = 0.1$  until the fatigue crack had grown 0.4 in. Figure 2 shows four samples of crack growth curves and data supplied to the writer. Table 2 presents data on the circled experimental points from which the crack growth curves were drawn and Table 8 presents the data from these curves.

Since the load decreased somewhat during testing, load range values were computed for the beginning and end of the entire crack growth curve. These two values were averaged for computation of the stress intensity amplitude  $\Delta K$  used in analysis of the data. The  $da/dN$  values used were determined from tangents to the curves at a fixed "grown" crack length of 0.2 in., i.e., "a" in Figure 1 was 0.6 in.

Their resultant plot of  $\Delta K$  vs  $da/dN$  is shown in Figure 3 in comparison with other work they referenced.

## PRESENT TYPE OF ANALYSIS

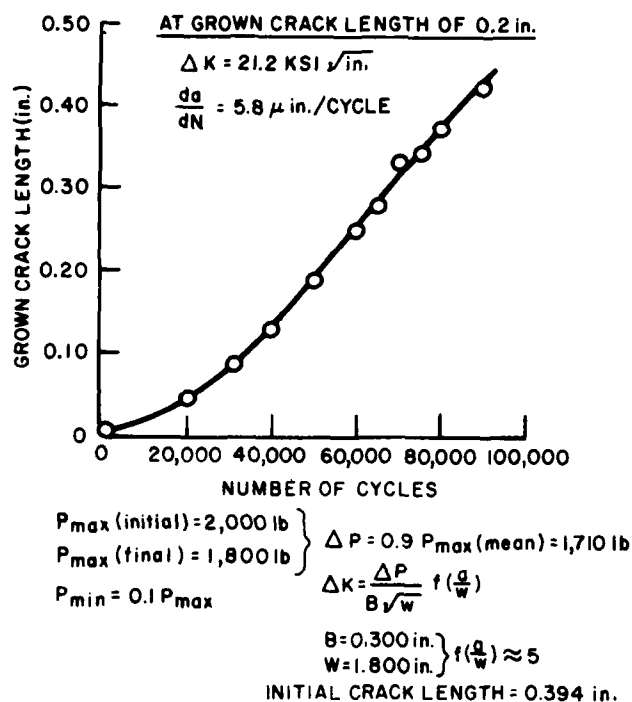
### General Considerations and Power Crack Growth Law

A postulated rate of crack growth equation is integrated here to give a straight line relationship between the integral and the number of cycles occurring between the limits of integration. If the form of rate equation is valid, values of the integral corresponding to observed total crack lengths plotted against the loading cycles to reach these crack lengths will lie on a straight line whose slope may be compared with a theoretical one. It is felt that this method is superior to the usual approach, that of trying to correlate with the unintegrated  $da/dN$  rate equation itself, largely because of difficulties in ascertaining satisfactory experimental values of  $da/dN$ . If  $da/dN$  is taken to be  $\Delta a/\Delta N$  where  $\Delta a$  is a small increment of crack growth, it is difficult to observe a suitable  $\Delta a$  for small  $\Delta N$  because of irregularity and lack of definition of the crack front. In fact,  $\Delta a$  may be different according to its observed location (sides of test piece or interior). Furthermore, it is difficult to obtain an appropriate rate from the slope of a curve drawn through selected  $da/dN$  data because of the diverse characteristics and, especially, lack of possible abrupt changes in slope, that apparently equally-suitable matching curves may have. In the present method only straight lines are run through the data; either the integrated rate equation is suitable for this or it must be changed. As will be shown, the rate at any crack length is very simply given by the one slope of the line through the range of matching data together with the value of  $\Delta K$  at that crack length.



**MATERIAL - B<sub>1</sub> (HEAT 2321, TYPE I)**

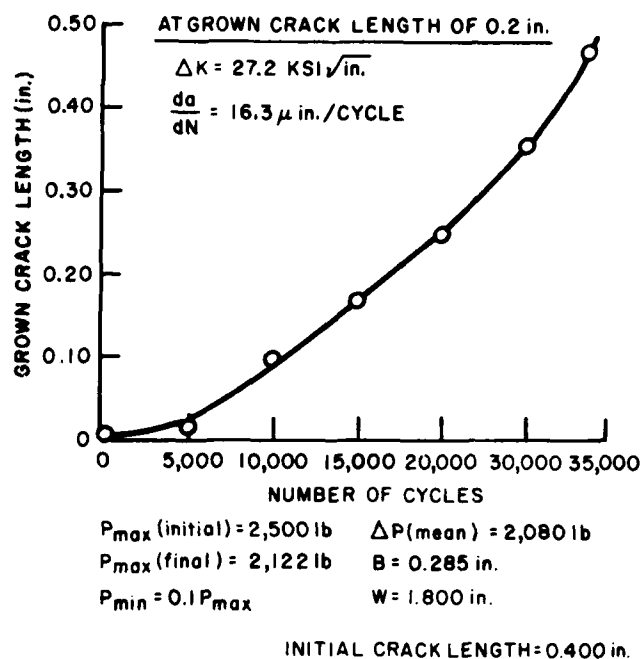
**SPECIMEN No.1**



(a)

**MATERIAL - B<sub>2</sub> (HEAT 2322, TYPE I)**

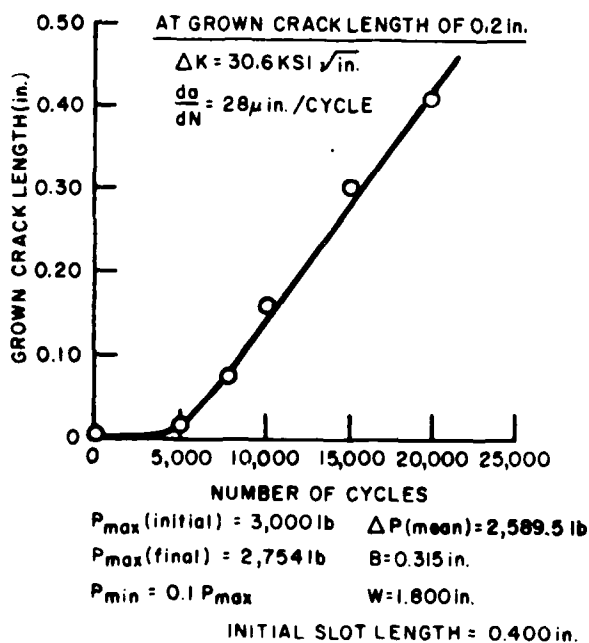
**SPECIMEN No.3**



(b)

**MATERIAL - B<sub>3</sub> (HEAT 2322, TYPE II)**

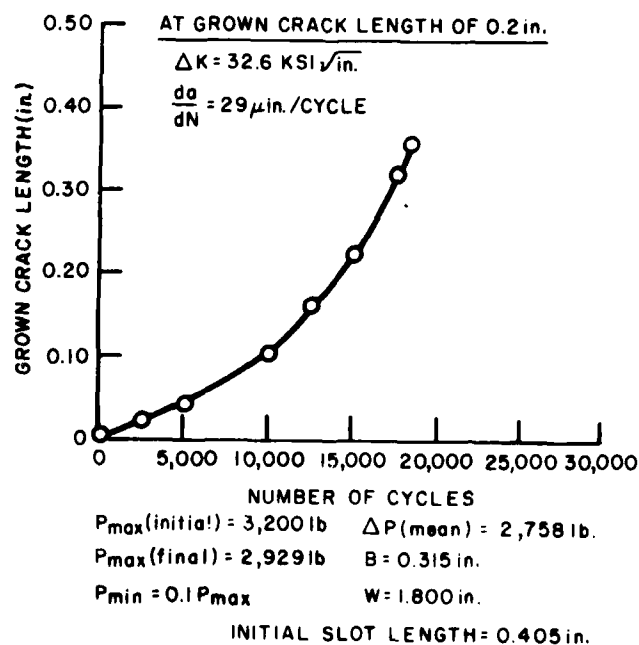
**SPECIMEN No.4**



(c)

**MATERIAL - B<sub>3</sub> (HEAT 2322, TYPE II)**

**SPECIMEN No.5**



(d)

Figure 2. Four samples of data sheets supplied by Lal-Weiss (3/8 X actual size).

Table 2. LAL-WEISS ORIGINAL TRIP STEEL DATA OF GROWN CRACK LENGTH (GCL) vs CYCLES (N) AND THE CORRESPONDING  $m = 2$  CTS INTEGRAL  $I_{CTS2}$  vs  $X \equiv a/w$  USING ASTM STANDARD E399-70T FORMULA FOR K

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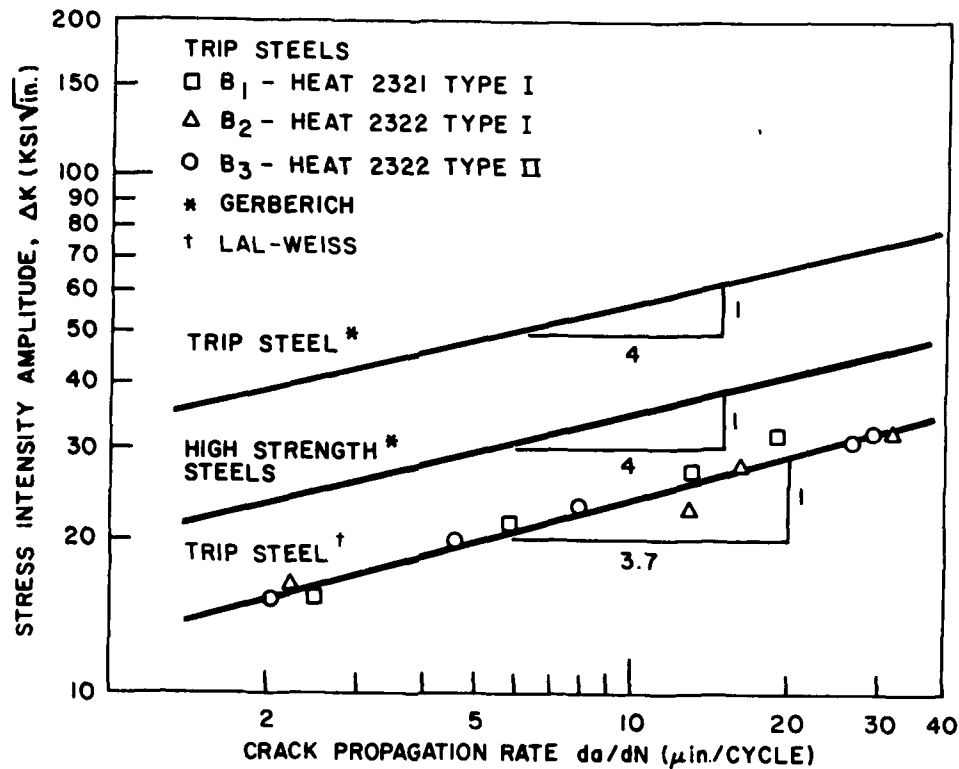


Figure 3. Log-log plot of fatigue crack growth rates  $da/dN$ , vs stress intensity amplitudes,  $\Delta K$ , of Lal-Weiss and Gerberich.<sup>1</sup>

Thus, consider Equation (1)

$$\frac{da}{dN} = C_m (\Delta K)^m \quad (1)$$

in which the constant<sup>†</sup>  $C_m$  evidently has the dimensions  $(\text{length})^{1-m/2} (\text{stress})^{-m}$ .

Equation (1) may also be written as

$$\frac{d(a/b)}{dN} = \left( \frac{\Delta K}{b^{1/2} \Delta S_n} \right)^m \left[ \frac{C_m b^{m/2} (\Delta S_n)^m}{b} \right] \quad (1a)$$

<sup>†</sup> Note that the stress dimension in  $C_m$  is not determined when  $C_m$  is from tests made with a single value of  $\Delta S_n$ . For example, unless otherwise determined, the stress in  $C_m$  might be assumed to be  $[E^{xY} (\Delta S_n)^{1-x-y}]$  where  $E$  is the Modulus of elasticity and  $Y$  is yield strength; thus, in this case  $da/dN$  is not really proportional to  $\Delta K$ .

1. GERBERICH, W.W. Metastable Austenitic Steel With Ultra High Strength and Toughness. SAE Paper No. 690262, Int. Auto. Eng. Cong., Detroit, Mich., January, 1969.

where  $b$  is an arbitrary length dimension that factors out along with the nominal loading stress range  $\Delta S_n$ , from the analytical expression for  $\Delta K$  for the test specimen geometry and loading. Thus,  $\Delta K/(b^{1/2} \Delta S_n)$  is a dimensionless function of  $(a/b)$  and dimensionless ratio constants that represent the test specimen geometry. (See, in particular, ASTM Standard E399-70T where we chose  $b = w$ .)

Hence,

$$I_{gm} \equiv \int_{(a/b)_0}^{(a/b)} \frac{d(a/b)}{\left[ \Delta K / (b^{1/2} \Delta S_n) \right]^m} = b^{m/2-1} C_m \int_{N_0}^N (\Delta S_n)^m dN \equiv b^{m/2-1} C_m \Sigma_m \quad (2)$$

so that if  $\Delta S_n$  is constant

$$I_{gm} = b^{m/2-1} C_m (\Delta S_n)^m (N - N_0). \quad (2a)$$

Thus, if  $C_m$  is constant,  $I_{gm}$  is rectilinearly ("straight line") related to  $\Sigma_m$  or, more simply, if  $\Delta S_n$  is also constant, to  $N$ .

We characterize here the integral on the right by  $\Sigma_m$  and the integral on the left by  $I_{gm}$ , where  $g$  stands for the geometry of the test specimen and  $m$  for the power of the rate relation, Equation (1). Thus,  $I_{CTS2}$  is the integral for the CTS with  $m = 2$ . The lower limit  $(a/b)_0$  can have any value so long as it is below the experimental range of  $(a/b)$ . We have provided tables of  $I_{CTS2}$  and  $I_{CTS4}$  with  $(a/b)_0 \equiv x_0 = 0.2$  for use with the CTS.

For a very simple example of  $I_{gm}$  consider the case of a central crack ( $g = cc$ ) in a wide plate under uniform tensile load perpendicular to the crack and far from it, with  $m = 2$ . Then,

$$\Delta K = \Delta S \sqrt{\pi a}$$

so that

$$I_{cc2} = \int_1^{a/a_0} \frac{da/a_0}{(\sqrt{\pi a/a_0})^2} = (\ln a/a_0)/\pi \quad (3)$$

Here we have chosen  $b = a_0$  to be the initial crack length.

Note that

$$\left[ \frac{1}{(N - N_0)} \int_{N_0}^N (\Delta S_n)^m dN \right]^{1/m} \equiv \left[ \Sigma_m / (N - N_0) \right]^{1/m} \quad (4)$$

is a constant stress range equivalent to the actual spectrum  $\Delta S_n$  in  $N - N_0$  cycles of loading.

It was evident from Equation (2) that  $I_{gm}$  is rectilinearly related to  $\Sigma$ , or more simply to  $N$  where  $\Delta S_n$  is constant. Suppose values of  $I_{gm}$  corresponding to experimental values of  $x = a/b$  were plotted against values of  $\Sigma$ , or  $N$ , corresponding to  $a/b$  and found to conform to a straight line over some range. Then, in that range,

$$C_m = \frac{(I_{gm})_2 - (I_{gm})_1}{(\Sigma_m)_2 - (\Sigma_m)_1} \cdot \frac{1}{b^{m/2-1}} \quad (5)$$

or, where  $\Delta S_n$  is constant,

$$\begin{aligned} C_m &= \frac{(I_{gm})_2 - (I_{gm})_1}{N_2 - N_1} \cdot \frac{1}{b^{m/2-1} (\Delta S_n)^m} \\ &= \text{Slope} / \left[ b^{m/2-1} (\Delta S_n)^m \right], \end{aligned} \quad (5a)$$

where  $N_2$ ,  $(I_{gm})_2$  and  $N_1$ ,  $(I_{gm})_1$  are any two points on the line and  $[(I_{gm})_2 - (I_{gm})_1] / (N_2 - N_1)$  is its slope. Thus,  $C_m$  is determined from the data and we see that a straight line plot of  $I_{gm}$  vs.  $N$  along with Equation (5) or (5a) completes the match to Equation (1).

From this line the rate of growth for any value of  $\Delta K$  is then found by substituting  $C_m$  from Equation (5) or (5a) into Equation (1), e.g.,

$$\frac{da}{dN} = b(\text{Slope}) \left[ \Delta K / (b^{1/2} \Delta S_n) \right]^m, \quad (6)$$



where  $\Delta S_n$  is constant or the constant equivalent to the actual spectrum in the region of constant slope.

Experience has shown that  $I$  vs  $N$  curves are often made up of straight line segments with different slopes. Thus,  $\ln da/dN$  vs  $\ln \Delta K$  curves will, in accordance with Equation (6), be made up of straight line segments with a jump in  $da/dN$  at the end of each segment. There appears to be a tendency for these transitions to be spaced at equal intervals of  $\ln \Delta K$  so that the ratio of rates at the transitions as well as of the  $\Delta K$ 's at two successive transitions is constant. In this case the transition points are connected by two parallel straight line envelopes (Figure 4). There also seems to be a tendency for the segments to lie between these two envelopes even where the transitions do not occur at exactly equal intervals of  $\Delta K$ .

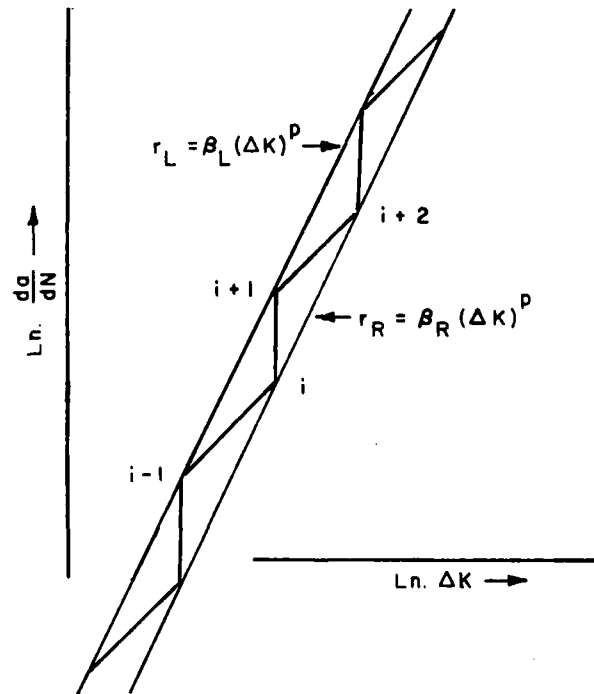


Figure 4. Idealized  $da/dN$  vs  $K$  Diagram.

In Figure 4 let  $da/dN$  on the right-hand envelope be  $r_R$  and along the left-hand envelope be  $r_L$ . Then,

$$r_R = \beta_R (\Delta K)^p$$

$$r_L = \beta_L (\Delta K)^p ,$$

where  $p$  and the  $\beta$ 's are constant.

Let subscript  $i$  denote a specific transition and let  $C_{i-1,i}$  be  $C_m$  of Equation (1), between  $i$  and  $i-1$ . Then,

$$r_{R,i} = C_{i-1,i} (\Delta K)_i^m = \beta_R (\Delta K)_i^p \quad (7a)$$

$$r_{L,i-1} = C_{i-1,i} (\Delta K)_{i-1}^m = \beta_L (\Delta K)_{i-1}^p , \quad (7b)$$

so that the envelope on the right is

$$r_R = C_{i-1,i} (\Delta K)_i^{m-p} (\Delta K)^p \quad (8a)$$

and the envelope on the left is

$$r_L = C_{i-1,i} (\Delta K)_{i-1}^{m-p} (\Delta K)^p . \quad (8b)$$

From Equations (7a) and (7b) the ratio of  $\beta$ 's as well as the ratio of rates at the end and beginning of a segment are constant since the ratios of the corresponding  $(\Delta K)$ 's are constant, i.e.,

$$\frac{\beta_R}{\beta_L} = \left[ \frac{(\Delta K)_i}{(\Delta K)_{i-1}} \right]^{m-p} \quad \text{and} \quad \frac{r_{R,i}}{r_{L,i-1}} = \left[ \frac{(\Delta K)_i}{(\Delta K)_{i-1}} \right]^m . \quad (9)$$

At  $i$ ,  $(\Delta K) = (\Delta K)_i$  so that from Equations (8a) and (8b) the jump rate ratio is

$$\frac{r_{L,i+1}}{r_{R,i}} = \left[ \frac{(\Delta K)_i}{(\Delta K)_{i-1}} \right]^{p-m} . \quad (10)$$

This is also the ratio of the  $C$ 's on two successive segments, i.e.,

$$\frac{C_{i+1,i+2}}{C_{i-1,i}} = \frac{r_{L,i+1}}{r_{R,i}} = \left[ \frac{(\Delta K)_i}{(\Delta K)_{i-1}} \right]^{p-m} = \frac{\beta_L}{\beta_R} . \quad (11)$$

Finally, the ratio of the jump rate ratio to the ratio of rates at the end and beginning of the segment ending at the jump is

$$(r_{L,i+1}/r_{R,i})/(r_i/r_{i-1}) = \left[ \frac{(\Delta K)_i}{(\Delta K)_{i-1}} \right]^{p-2m}, \quad (12)$$

which is a constant.

Analyses of many experiments suggest that  $m = 2$  and  $p = 4$  and that the ratio of rates at the end and beginning of any  $m = 2$  straight line segment spanning the distance between the envelope lines of a plot of  $\log da/dN$  vs  $\log (\Delta K)$  is two as specified in the generalized Beeuwkes crack growth law discussed in the next section. For such a case we consider a segment whose equation is known from theory or experiment and which begins at a known value  $(\Delta K) = (\Delta K)_{i-1}$ , i.e.,

$$\frac{da}{dN} = C_{i-1,i} (\Delta K)^2 \quad \text{with} \quad (\Delta K)_{i-1} \leq (\Delta K) \leq (\Delta K)_i, \quad (13)$$

where  $C_{i-1,i}$  and  $(\Delta K)_{i-1}$  are known. In this case we will now find the equations of the envelopes and the characteristics of the actual growth.

From Equations (7a) and (8a)

$$2 = \frac{r_{R,i}}{r_{L,i-1}} = \left[ \frac{(\Delta K)_i}{(\Delta K)_{i-1}} \right]^2 = \frac{\beta_R}{\beta_L} \quad (14)$$

so that

$$(\Delta K)_i = \sqrt{2} (\Delta K)_{i-1}. \quad (15)$$

Hence from Equations (8a) and (8b) the rate equations of the envelopes are

$$r_L = C_{i-1,i} (\Delta K)_{i-1}^{-2} (\Delta K)^4 \quad (16a)$$

$$r_R = C_{i-1,i} (\Delta K)_{i-1}^{-2} (\Delta K)^{4/2}, \quad (16b)$$

$$\text{so that } \log r_L - \log r_R = \log 2. \quad (17)$$

The ratio of the rate constants of two adjacent growth segments is

$$\frac{C_{i+1,i+2}}{C_{i-1,i}} = \frac{r_{L,i+1}}{r_{R,i}} = \frac{\beta_L}{\beta_R} = 2 \quad . \quad (18)$$

The ratio of the jump in rate from one segment to the next to the increase in rate from the beginning to the end of the first segment is

$$(r_{L,i+1}/r_{R,i})/(r_1/r_{i-1}) = \left[ \frac{(\Delta K)_i}{(\Delta K)_{i-1}} \right]^{4-2(2)} = 1 \quad . \quad (19)$$

#### Beeuwkes Crack Growth Law

This crack growth law for applied stress amplitudes\* (well below the yield strength) and for plane strain, the present case, was originally written

$$\frac{da}{dN} = \left( \frac{2}{3} \right)^{1/3} (1 - \mu^2)^{4/3} \left( \frac{S}{E} \right)^2 \left( \frac{E}{Y} \right)^{2/3} a \quad (20)$$

and may be written

$$\frac{da}{dN} = \frac{C_B}{\pi} \left( \frac{S}{E} \right)^2 \left( \frac{E}{Y} \right)^{2/3} \left( \frac{K_1}{S} \right)^2 \quad , \quad (20a)$$

where

$$C_B = \left( \frac{2}{3} \right)^{1/3} (1 - \mu^2)^{4/3}$$

and where  $\mu$  is Poisson's ratio,  $E$  is the Young's Modulus of Elasticity,  $Y$  is the yield strength, and  $S$  is the "nominal field" stress,\* i.e., the stress equivalent to the nominal loading stress in a long, wide tension specimen having a small, central crack of length  $2a$  lying perpendicular to the loading direction, i.e.,  $S = K_1^2/\pi$  in modern stress intensity factor notation.† In the case of considerable

\*For sharp notches other than the small central crack across the tension field,  $S$ , in a wide plate,  $S = [k_g/(2\sqrt{a/\rho})] S_{gn}$  as  $\rho \rightarrow 0$ , where  $k_g$  is the elastic stress concentration factor for the geometry  $g$  and  $S_{gn}$  is the corresponding nominal stress for that geometry ( $k_g$  includes an  $a/\rho$  factor).

†Taking as definition,  $K = S\sqrt{\pi a}$  in the case of the above tension member with the central crack.

plasticity,  $S/E$  was to be regarded as a loading strain. The effect of different ranges of stress had not been worked out.

Subsequent analyses of experiments found in the literature indicated that:

(1) at least for tension-tension stressing  $(S_{\max} - S_{\min}) \equiv \Delta S_n$  could be substituted for  $S$  so that  $(\Delta K)^2/\pi$  could be substituted for  $S^2 a$ ; (2) that at the start of most of the tests the constant  $da/dN/(\Delta K)^2$  was uniformly distributed within  $1/\sqrt{2}$  to  $\sqrt{2}$  times  $C_B$  in Equation (20a); (3) that though the data could be well-matched assuming the rate remained proportional to  $(\Delta K)^2$  for constant  $(S_{\max} - S_{\min})$ , the constant of proportionality,  $C_B$ , changed abruptly after various amounts of crack growth, sometimes by a slight amount, but where the change was substantial,\* by factors averaging close to two and this change was not typically associated with a transition from flat to shear mode; and, finally, (4) that though the length of crack at changes was variable even under similar conditions of testing, there was a tendency, in plots of  $\log da/dN$  vs  $\log \Delta K$ , for the second-power straight-line segments matched with the substantially different constants of proportionality, to lie within two-fourth-power straight line envelopes.

In the latter case the ratio

$$(\Delta K_{ne})^2/(\Delta K_{no})^2 = 2 \quad ,$$

where  $ne$  corresponds to the end and  $no$  to the beginning of segment "n". With  $\Delta S$  constant this is  $(a_{ne}/a_{no}) = 2$ , the ratio of crack lengths at the beginning and end of any segment, and the growth is thus in conformance with the formulae at the end of the preceding section, Equations (13) to (19).

For short cracks the uncertainties in crack length determination are sufficient to make the length of the line segments  $(a_{ne} - a_{no}) = a_{no}$  indistinguishable so that the experimental law of growth appears to be that of the fourth power of  $\Delta K$  coupled with (i.e., data within) a scatter band. For longer crack lengths however, cases have come to the present writer's attention where a law of growth has been suggested on the basis of experimental data that seems to him to actually correspond to enclosing envelopes, the experimental points of individual tests making up the whole plot appearing to correspond to a lesser slope than that of such envelopes.

Recapitulating, with  $\Delta S_n \equiv S_{\max} - S_{\min}$ , where  $S$  represents nominal applied loading stress, we would expect

$$\frac{da}{dN} = \frac{C_B}{\pi} \left( \frac{\Delta S_n}{E} \right)^2 \left( \frac{E}{Y} \right)^{2/3} \left( \frac{\Delta K}{\Delta S_n} \right)^2 \quad , \quad (21)$$

\* Higher rates are natural if cleavage areas occur at the  $\Delta K$  used, as has been observed in some aluminum tests, but there are other possible mechanisms for the rate transitions. For example, the writer has suggested that growth may be concentrated in statistically uniformly distributed points along the crack front so that at a transition to a higher rate a growth point in between each is added, thus doubling the rate.

where the constant  $C_B$  introduced here is expected to have values given by

$$2^n / \sqrt{2} \leq C_B / \left[ (2/3)^{1/3} (1 - \mu^2)^{4/3} \right] \leq 2^n \sqrt{2} \quad (22)$$

instead of simply\*

$$C_B = (2/3)^{1/3} (1 - \mu^2)^{4/3} \cong \pi/4 ,$$

the value of  $C_B$  in Equation (20a).

Here  $n = 0$ , basically but there may be transitions with  $n$  having the values

$$\dots, -2, -1, 0, 1, 2, \dots \quad (23)$$

If  $n = 0$ , we have approximately

$$\pi/(4\sqrt{2}) \cong 0.555 \leq C_B \leq 1.111 \cong \pi\sqrt{2}/4 \quad (24)$$

In Equation (21), the term  $(\Delta K/\Delta S_n)$  is the analytical expression for  $\Delta K$  with the nominal stress factored out.

## SECOND POWER CTS INTEGRAL AND ANALYSIS; $\Delta K$ BY ASTM STANDARD E399-70T

### Constant Loading Range

$\Delta K$  for a CTS can be calculated from the power series law given in ASTM Standard E399-70T. It is

$$\Delta K = (\Delta P/BW^{1/2})(x^{1/2})(29.6 - 185.5x + 655.7x^2 - 1017x^3 + 638.9x^4) \quad (25)$$

---

\*  $n$  was chosen to put scatter in experimental determination of this basic value  $C_B$  to either side of  $C_B$ . If  $n$  were replaced by  $(n + 1/2)$  the scatter would be to one side:

$$2^n \leq C_B / \left[ (2/3)^{1/3} (1 - \mu^2)^{4/3} \right] \leq 2^{n+1}$$

so that for the basic form,  $n = 0$ ,

$$1 \leq C_B / \left[ (2/3)^{1/3} (1 - \mu^2)^{4/3} \right] \leq 2 ,$$

where with  $a$  = distance from the loading axis to the crack front (i.e.,  $a$  = ICL + grown crack length), Figure 1,

$$x = a/w$$

and

$$\Delta P/BW = \Delta S_n \quad . \quad (26)$$

The second power CTS integral expression corresponding to this formula for  $\Delta K$  may be gotten from Equation (2), i.e.,

$$I_{CTS2} \equiv \int_{x_0}^x \frac{dx}{x(29.6 - 185.5x + 655.7x^2 - 1017x^3 + 638.9x^4)^2} \quad . \quad (27)$$

The integral was calculated and tabulated along with  $I_{CTS2}$  from other proposed expressions for  $K$  to be discussed later, page 30, Table 7, briefly in Table 3 and more fully in Appendix A for the range

$$0.2 \leq x \equiv a/w \leq 0.8 \quad .$$

The lower bounding limit on  $x$  was selected because, for CTS,  $K$  for cracks for  $x \equiv a/w < 0.2$  is too sensitive to the actual distribution of applied load to warrant general formulation. (Thus, experimental determination of  $K$  for  $x < 0.2$  for each particular loading pin configuration being used is desirable.) The upper limit was chosen to avoid getting into a region of side edge effects. The integral is plotted in Figure 5.

Also, a Pade approximant<sup>2</sup> was approximately fitted to the tabulated results. It is

$$I_{CTS2} \cong - \frac{1 - 3.400x - 8.030x^2}{1 - 1.364x + 7.040x^2} \times 5.548 \times 10^{-3} \quad .$$

Using Appendix A to get  $I_{CTS2}$  from  $a/w$  values,  $I_{CTS2}$  vs  $N$  plots were made using the actual "grown" crack length vs cycle  $N$  data points (Table 2), as well as data points taken at regular intervals from smooth curves drawn by Lal-Weiss through the actual data points (Figure 2).

2. BAKER, G. A. JR. Accurate Long-Range Extrapolation - The Pade Approximant.  
Brookhaven National Laboratory, Brookhaven Lecture Series, BNL 50241 (T576),  
Associated Universities, Inc., Contract with U.S. Atomic Energy Commission.

Table 3. CTS INTEGRAL  $I_{CTS2} \times 10^6$  vs  $X \equiv a/w$  FOR ASTM STANDARD E399-70T, SRAWLEY, BOWIE, and BEEUWKES  
 $K$  vs  $X \equiv a/w$ . SECOND-DEGREE PADE USED FOR BEEUWKES AND BOWIE  $K$ , THIRD DEGREE PADE FOR SRAWLEY  $K$ .  
 [SEE EQUATION (27), FIGURE 5, AND TABLE 7.] CONDENSED TABLE.

X	STANDARDS	SRAWLEY	BOWIE	BEEUWKES	X	STANDARDS	SRAWLEY	BOWIE	BEEUWKES
.20	0000	0000	0000	0000	.50	7214	7180	8172	8156
.21	0369	0440	0480	0516	.51	7319	7280	8278	8258
.22	0734	0858	0943	1009	.52	7418	7374	8377	8354
.23	1094	1257	1389	1478	.53	7511	7462	8469	8444
.24	1448	1637	1818	1924	.54	7599	7546	8555	8528
.25	1794	1999	2230	2349	.55	7680	7624	8635	8607
.26	2132	2344	2627	2754	.56	7756	7697	8710	8681
.27	2461	2674	3007	3140	.57	7827	7765	8779	8749
.28	2780	2989	3372	3506	.58	7893	7829	8843	8813
.29	3088	3289	3721	3855	.59	7955	7888	8902	8872
.30	3386	3575	4056	4187	.60	8011	7944	8957	8928
.31	3673	3849	4376	4502	.61	8064	7995	9008	8979
.32	3948	4110	4682	4801	.62	8112	8042	9055	9026
.33	4213	4360	4974	5086	.63	8156	8086	9098	9070
.34	4466	4598	5252	5356	.64	8197	8126	9137	9110
.35	4708	4825	5517	5612	.65	8234	8163	9173	9147
.36	4940	5042	5769	5855	.66	8268	8196	9206	9181
.37	5161	5249	6009	6086	.67	8299	8227	9236	9212
.38	5372	5446	6237	6304	.68	8327	8255	9264	9240
.39	5572	5634	6453	6511	.69	8353	8280	9289	9266
.40	5764	5813	6658	6707	.70	8375	8303	9311	9290
.41	5945	5983	6853	6892	.71	8396	8324	9332	9311
.42	6118	6146	7036	7067	.72	8415	8342	9350	9330
.43	6283	6300	7210	7233	.73	8431	8359	9366	9347
.44	6438	6446	7373	7389	.74	8446	8373	9381	9363
.45	6586	6586	7528	7537	.75	8460	8386	9395	9376
.46	6726	6718	7673	7676	.76	8472	8398	9406	9389
.47	6859	6843	7810	7807	.77	8483	8408	9417	9399
.48	6984	6962	7938	7930	.78	8492	8416	9426	9409
.49	7102	7074	8059	8047	.79	8501	8424	9434	9417
.50	7214	7180	8172	8156	.80	8508	8430	9441	9424

It was found that the  $I_{CTS2}$ ,  $N$  points were lined up on one or more straight line segments<sup>3</sup> having different slopes, even for data taken from the smooth curves where the transitions in slope corresponding to the change from one straight line segment to another was not apparent.

Despite the scatter in the actual data there was no substantial difference in the straight line representations made with the two data sources. Thus, in the interest of brevity and economy only those plots made with the actual data points are reproduced in Figures 6 through 18.

Since the plots are made up of one or more straight line segments, the crack growth is correspondingly represented as increasing as the square of the crack tip stress intensity range,  $\Delta K$ , with a proportionality constant that changes abruptly, but only when growth shifts from any segment to another.

The constants  $C_B$  (previous section) derived from Figures 6 through 18, as well as those derived from the smooth curve data not reproduced here are shown in the accompanying Table 4. Considering irregularities in the data and other factors to be discussed later, the agreement with present type of analysis is considered satisfactory.

3. TREMBLAY, R.J. Fatigue Crack Growth Transitional Behavior. Army Materials and Mechanics Research Center, AMMRC TN 73-13, October 1973. For copies of this Technical Note, contact Dr. Beeuwkes, AMMRC, 923-5744.



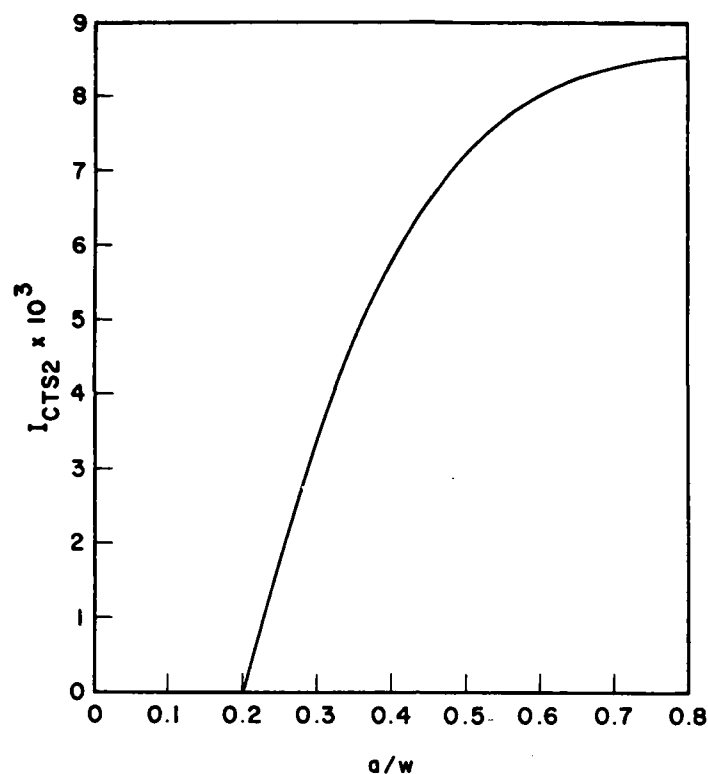


Figure 5. CTS Integral  $I_{CTS}$  vs  $a/w$  based on ASTM Standard E399-70T formula for  $K$ , and constant loading range.

### Effect of Gradual Decrease in Load During Testing

The matching formulae used to this point are designed for constant maximum and minimum loads. Actually, these loads decreased to around seven-eighths of their initial values during the testing, and in applying the previous analyses, the mean of the initial and final values were used. (This mean was also used by Syracuse University in its analysis.) The error introduced by this approximation was evaluated as follows, and so found to be essentially negligible. The actual load range\* was taken to be

$$\Delta P = \Delta P_{\text{mean}} (-0.6 a/w + 1.198) \quad (29)$$

with

$$0.22 \leq a/w \leq 0.44$$

---

\*Despite considerable scatter. (See  $r$  in Table 4.)

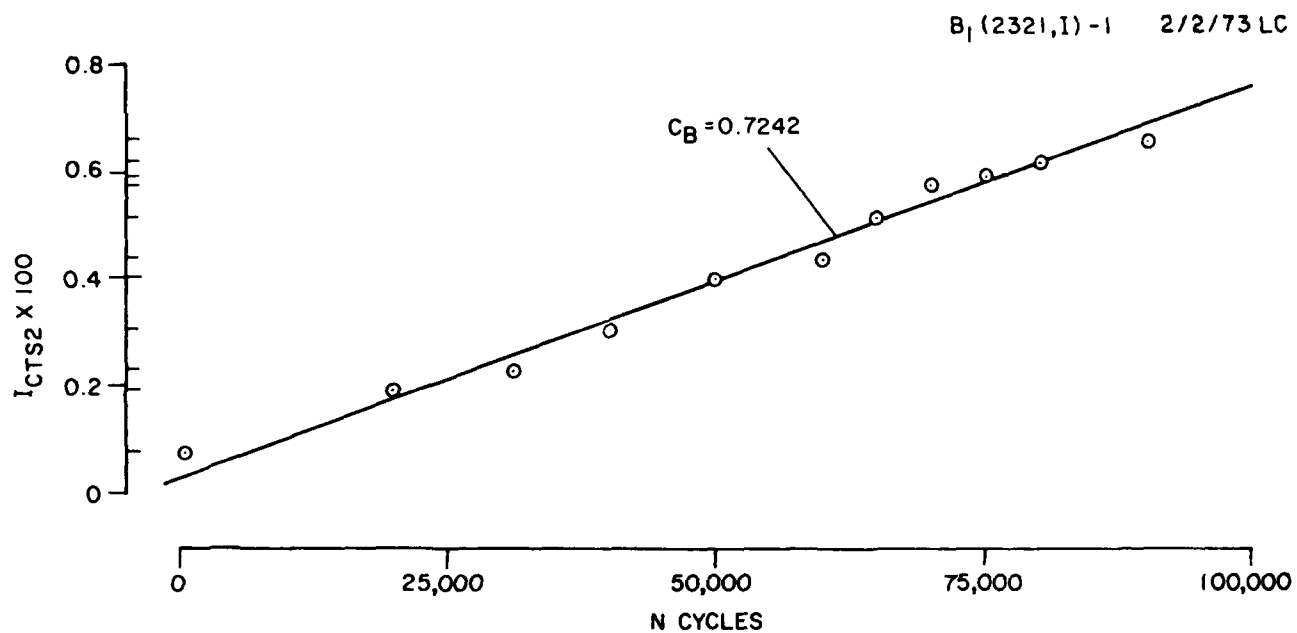


Figure 6.  $I_{CTS2}$  vs N for material B<sub>1</sub>, Type I, Specimen 1.

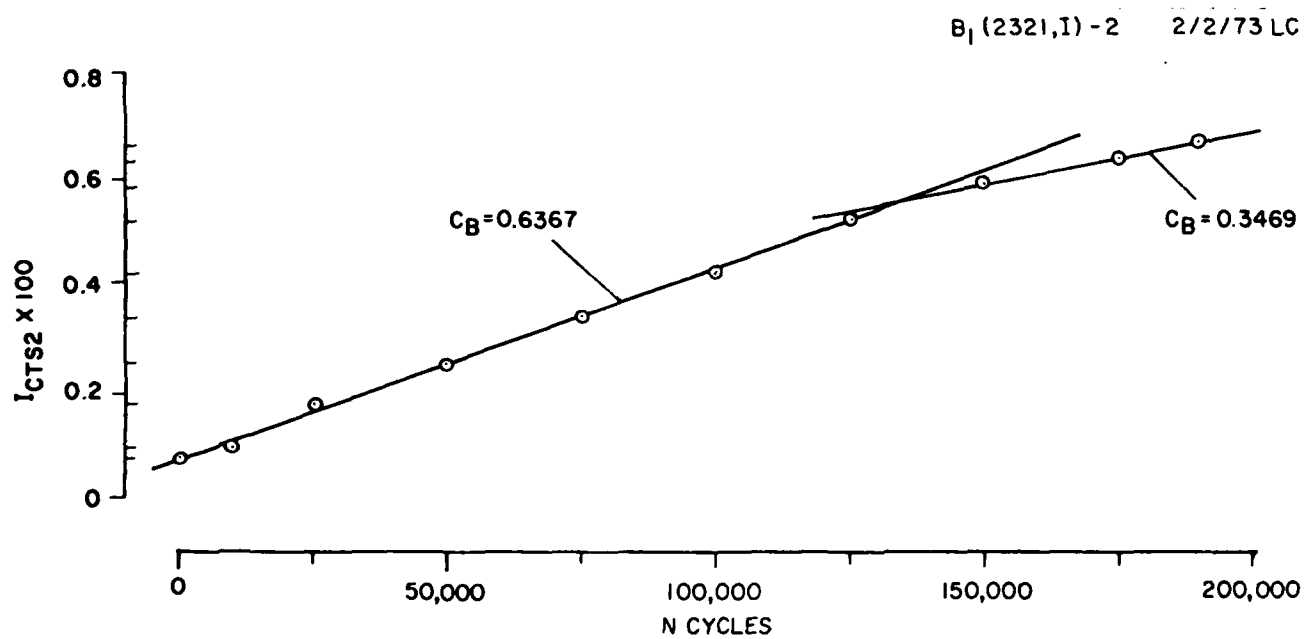


Figure 7.  $I_{CTS2}$  vs N for material B<sub>1</sub>, Type I, Specimen 2.

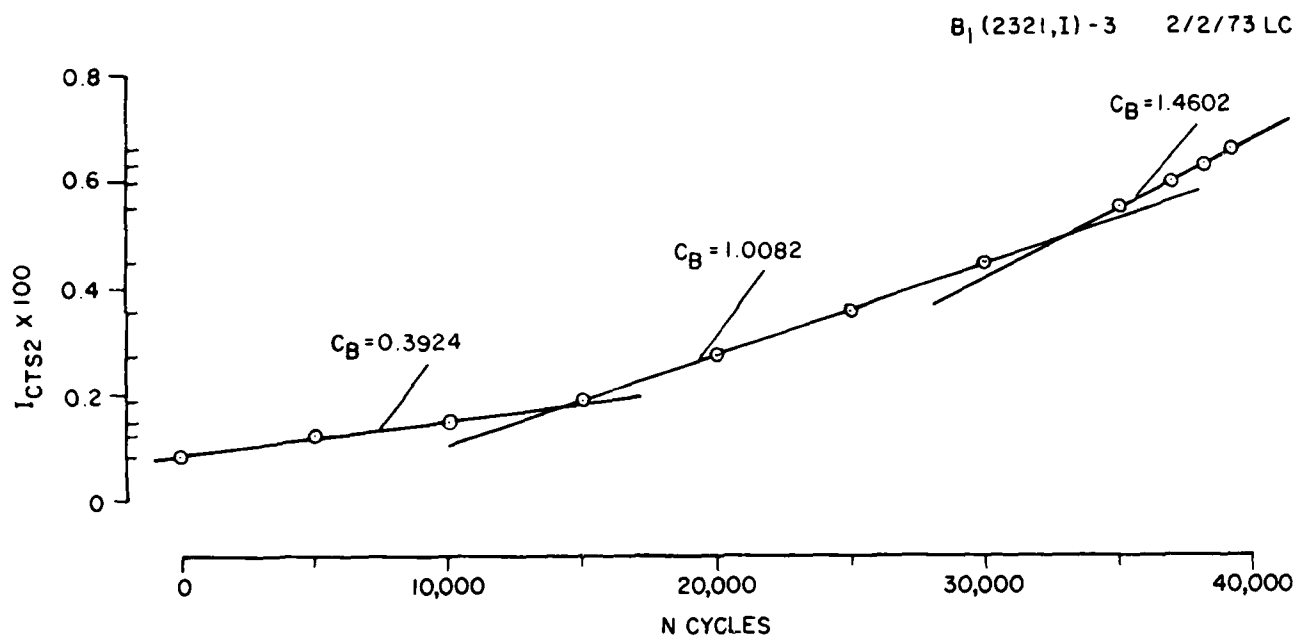


Figure 8.  $I_{CTS2}$  vs  $N$  for material B<sub>1</sub>, Type I, Specimen 3.

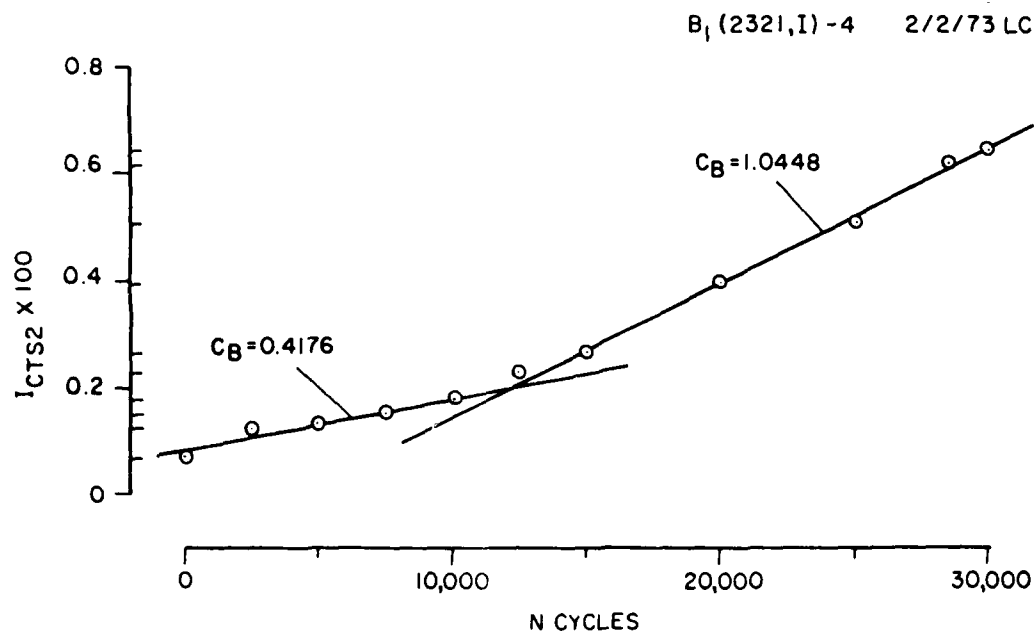


Figure 9.  $I_{CTS2}$  vs  $N$  for material B<sub>1</sub>, Type I, Specimen 4.

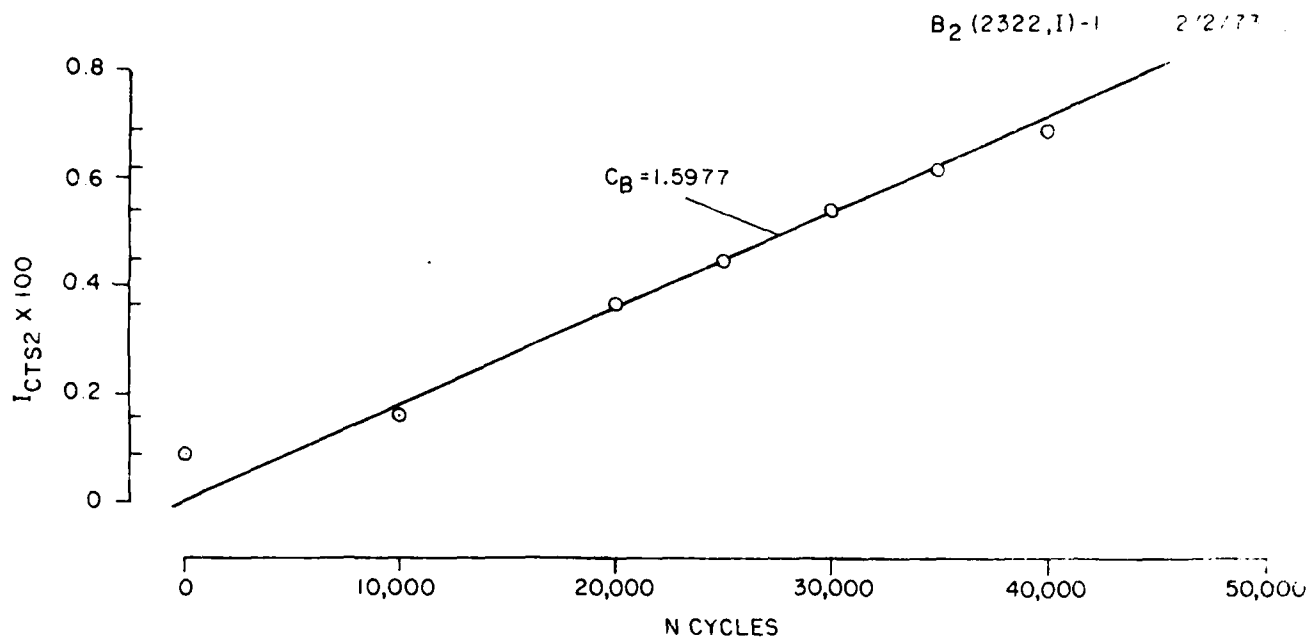


Figure 10.  $I_{CTS2}$  vs N for material B<sub>2</sub>, Type I, Specimen 1.

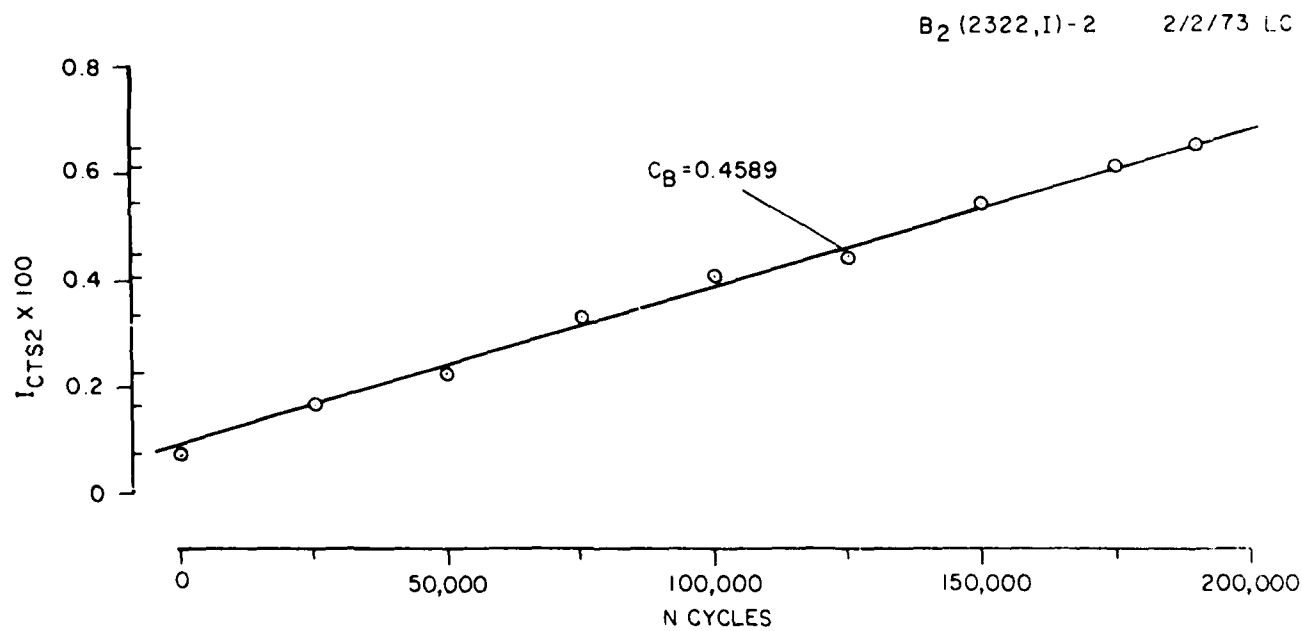


Figure 11.  $I_{CTS2}$  vs N for material B<sub>2</sub>, Type I, Specimen 2.

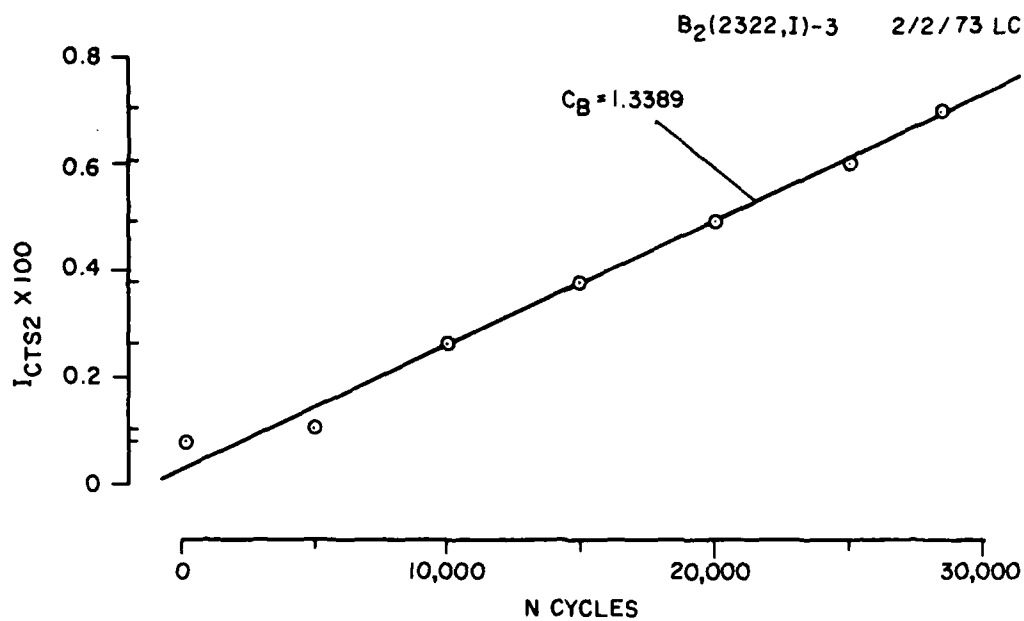


Figure 12.  $I_{CTS2}$  vs N for material B<sub>2</sub>, Type I, Specimen 3.

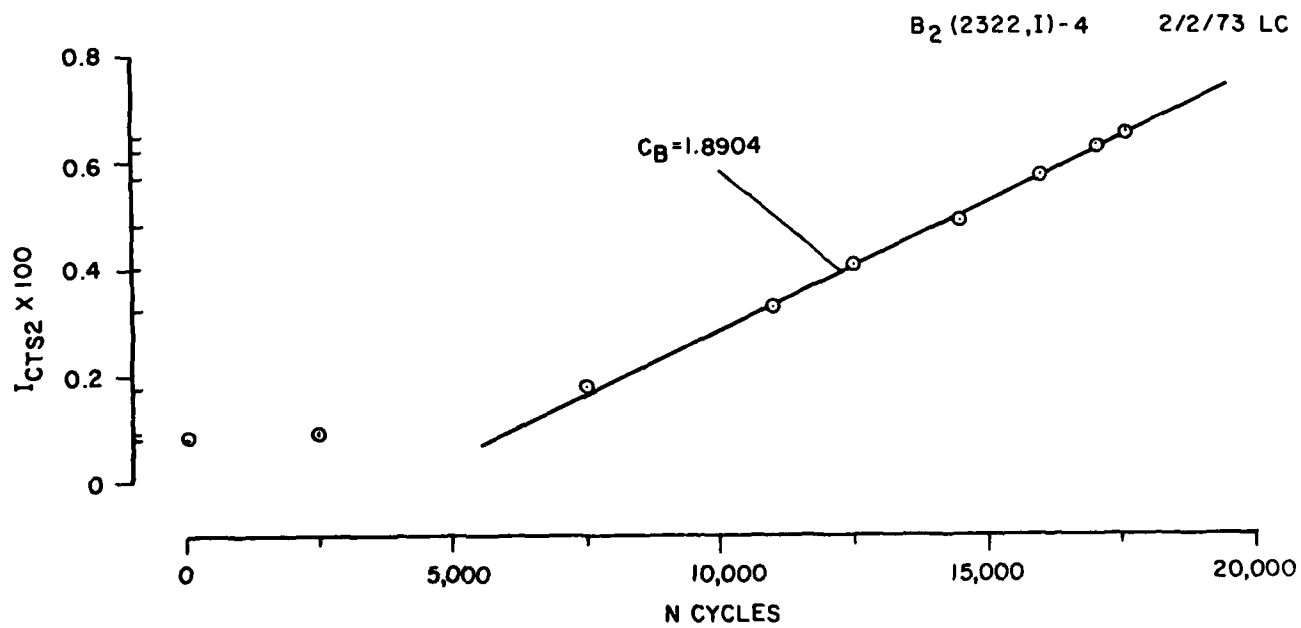


Figure 13.  $I_{CTS2}$  vs N for material B<sub>2</sub>, Type I, Specimen 4.

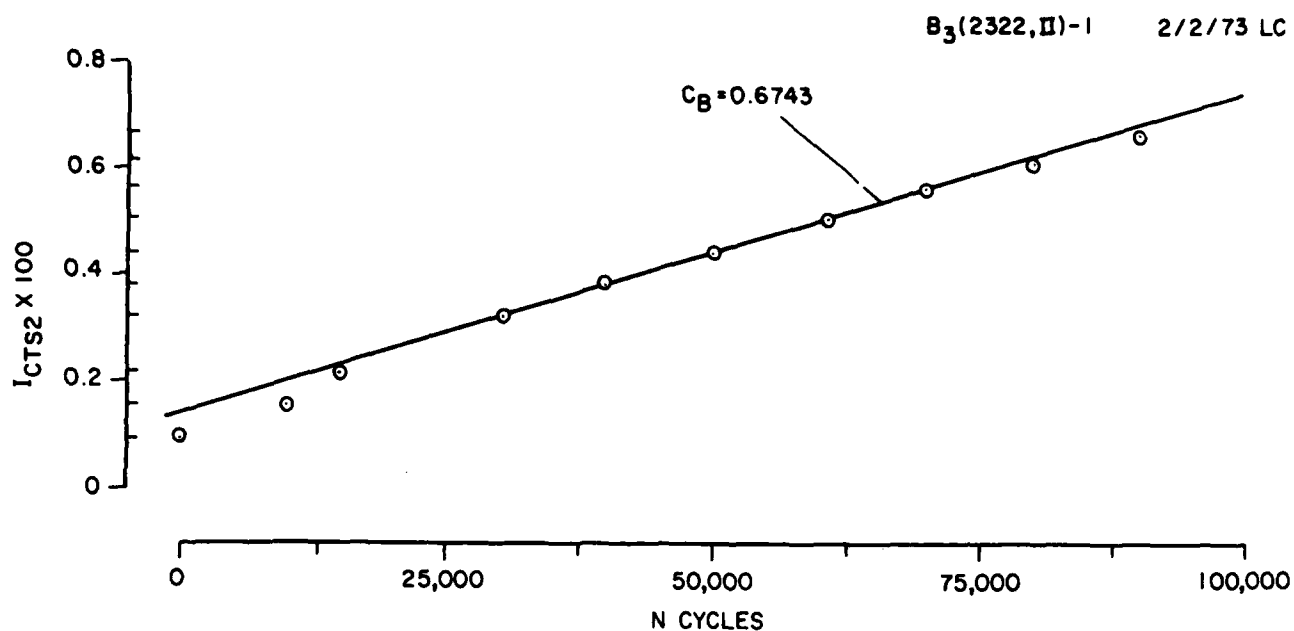


Figure 14.  $I_{CTS2}$  vs  $N$  for material  $B_3$ , Type II, Specimen 1.

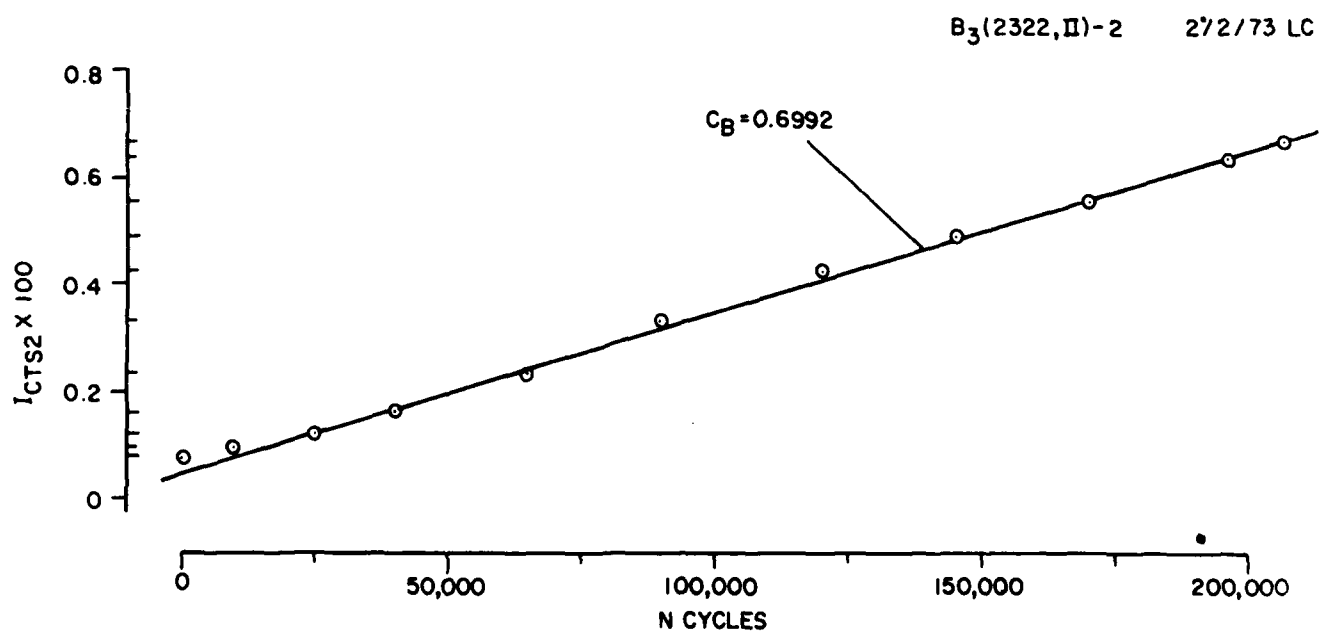


Figure 15.  $I_{CTS2}$  vs  $N$  for material  $B_3$ , Type II, Specimen 2.

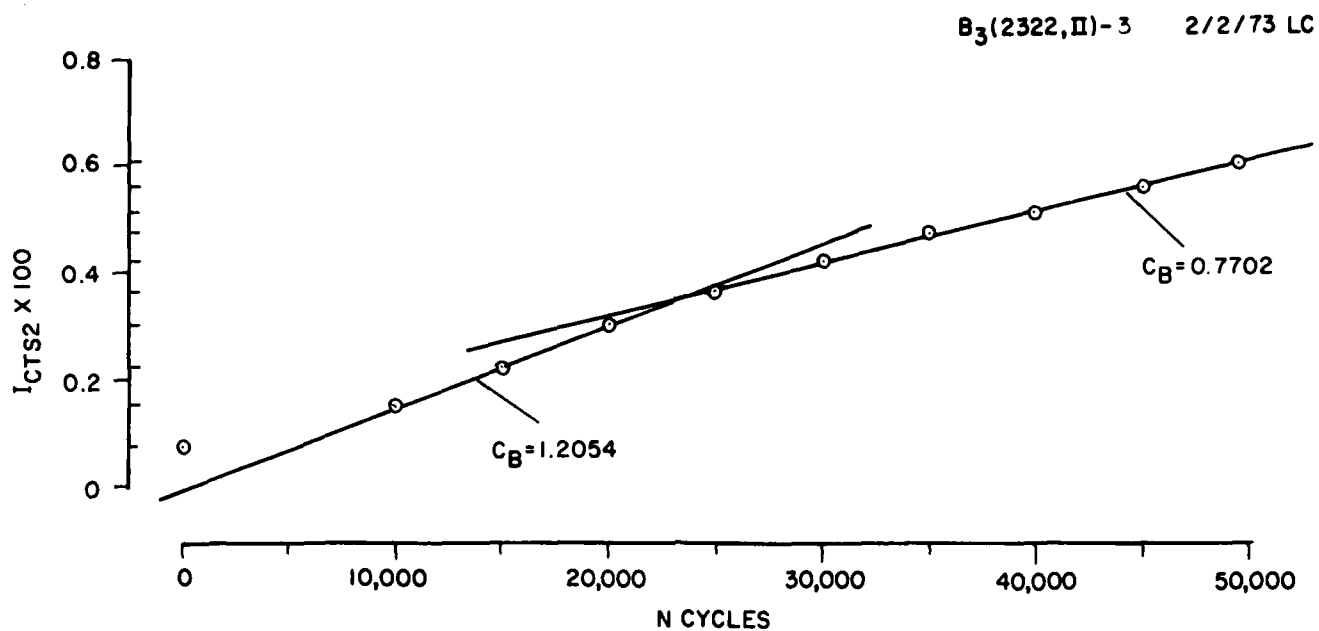


Figure 16.  $I_{CTS2}$  vs  $N$  for material  $B_3$ , Type II, Specimen 3.

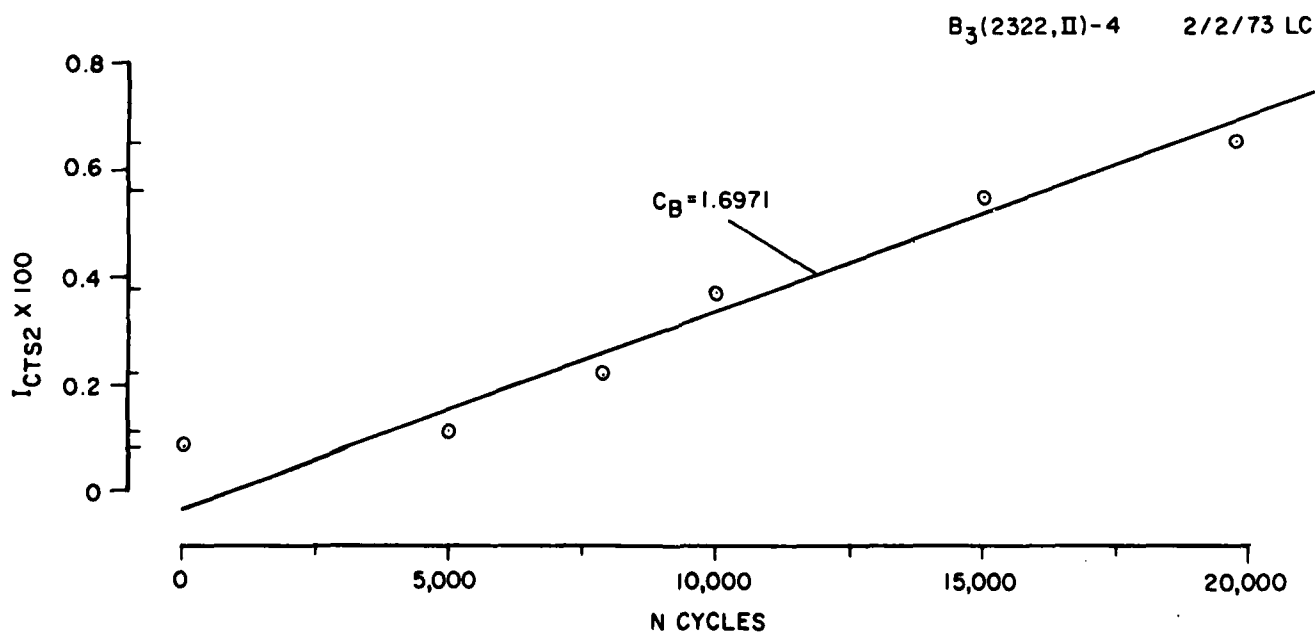


Figure 17.  $I_{CTS2}$  vs  $N$  for material  $B_3$ , Type II, Specimen 4.

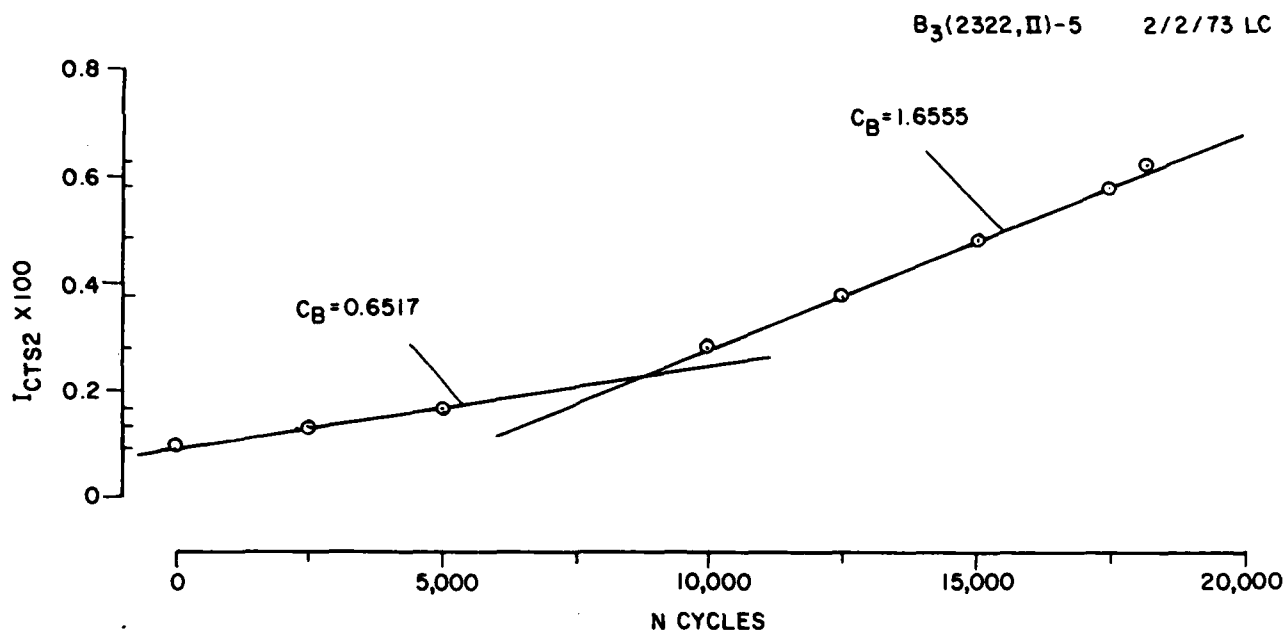


Figure 18.  $I_{CTS2}$  vs  $N$  for material B<sub>3</sub>, Type II, Specimen 5.

and hence the CTS integral of Equation (27) is modified to

$$I_{CTS2D} \equiv \int_{0.22}^x \frac{dx}{(-0.6x + 1.198)^2(x)(29.6 - 185.5x + 655.7x^2 - 1017x^3 + 638.9x^4)^2} + 0.734 \times 10^{-3} \quad (30)$$

and computer integrated to give Table 5 (condensed).  $0.734 \times 10^{-3}$ , the value of  $I_{CTS2}$  [Equation (27)] at  $x = 0.22$ , was added for the purpose of comparison, to make  $I_{CTS2} = I_{CTS2D}$  at  $x = 0.22$ .

Figure 19 compares the crack growth with the falling load (Table 5) with that under constant load. Both curves in Figure 19 correspond to a growth rate proportional to  $(\Delta K)^2$ . It appears that the difference between the two curves is within experimental error in the observed range of growth,  $0.22 \leq a/w \leq 0.44$ , and thus that the preceding analyses are valid. It is conceivable, however, that some of our  $I$  vs  $N$  plots of data taken with a falling load may appear to be made up of straight line segments with minor changes of slope instead of a single straight line.



Table 4. RATE PROPORTIONALITY CONSTANTS C DERIVED FROM FIGURES 6 THROUGH 18 AND CORRESPONDING FIGURES DERIVED FROM SMOOTH CURVES ON LAL-WEISS DATA SHEETS. (SEE PRESENT TYPE OF ANALYSIS.)

Material and Specimen	Max. Load		Mean Range $\Delta P$ , lb.	Fall-Off Ratio $r$	Slope No.	Original Data Points		Data Sheet Curves $C_B$
	Initial $P_{xi}$ , lb.	Final $P_{xf}$ , lb.				Slope $\times 10^8$ Figures 6-18	Growth Constant $C_B$	
B <sub>1</sub> I - 1	2000	1800	1710	0.10	1	7.46	0.7242	0.4737, 0.8969, 0.5892
B <sub>1</sub> I - 2	1500	1300	1260	0.13	1	3.56	0.6367	0.6544
					2	1.94	0.3469	0.4094
B <sub>1</sub> I - 3	2500	2375	2192	0.05	1	6.66	0.3934	0.7207
					2	17.07	1.0082	1.1165
					3	24.72	1.4602	1.4946
B <sub>1</sub> I - 4	3000	2782	2602	0.07	1	9.96	0.4176	0.4318
					2	24.92	1.0448	1.0733
B <sub>2</sub> I - 1	2000	1566	1685	0.22	1	18.38	1.5977	0.5728, 1.8078, 1.3037
B <sub>2</sub> I - 2	1500	1300	1260	0.13	1	2.952	0.4589	0.4569
B <sub>2</sub> I - 3	2500	2122	2080	0.15	1	23.474	1.3389	1.3746
B <sub>2</sub> I - 4	3000	2504	2477	0.16	1	--	--	0.3732
					2	47.00	1.8904	1.8984
E <sub>3</sub> II - 1	2000	1733	1680	0.13	1	6.079	0.6743	0.6256, 0.8153
B <sub>3</sub> II - 2	1500	1377	1295	0.08	1	3.745	0.6992	0.5377
B <sub>3</sub> II - 3	2500	1916	1987	0.23	1	15.21	1.2054	0.9113
					2	9.718	0.7702	
B <sub>3</sub> II - 4	3000	2755	2590	0.08	1	36.36	1.6971	0.4215, 2.2215, 1.3068
B <sub>3</sub> II - 5	3200	2929	2758	0.08	1	15.84	0.6517	0.8064
					2	40.24	1.6555	1.7485

NOTES: Min. Load = 0.1 Max. Load

$$\Delta P_{\text{mean}} = (\Delta P_{\text{initial}} + \Delta P_{\text{final}})/2$$

$$= (0.9P_{xi} + 0.9P_{xf})/2 = 0.45(P_{xi} + P_{xf})$$

$$\text{Range Fall-Off Ratio} \equiv r = (\Delta P_{\text{initial}} - \Delta P_{\text{final}})/$$

$$\Delta P_{\text{initial}}$$

$$= \text{Max. Load Fall-Off Ratio}$$

$$C_B = \frac{(\text{Slope})(\pi E^{4/3} \gamma^{2/3})(Bw)^2}{(\Delta P)^2}$$

For B<sub>1</sub> - 1:

$$C_B = \frac{(\text{Slope})(\pi)(28)^{4/3}(220)^{2/3}(0.3 \times 1.8)^2(10^{10})}{(1710)^2}$$

$$= \text{Slope} \left[ \frac{2839}{(1710)^2} \right] \times 10^{10} = 0.7242$$

with data from Table 1.

Table 5. CTS INTEGRAL  $I_{CTS2D}$  BASED ON ASTM STANDARD E399-70T FORMULA FOR  $R$  AND  $a$  DECREASING LOAD RANGE  $\Delta P = \Delta P_{\text{mean}} (-0.6 a/w + 1.98)$ . CONDENSED TABLE.

$x \equiv a/w$	$I_{CTS2D} - 0.000734$
0.220000	0
0.225000	0.000899
0.230000	0.001063
0.235000	0.001227
0.240000	0.001391
0.245000	0.001554
0.250000	0.001716
0.255000	0.001878
0.260000	0.002039
0.265000	0.002198
0.270000	0.002357
0.275000	0.002515
0.280000	0.002671
0.285000	0.002826
0.290000	0.002980
0.295000	0.003132
0.300000	0.003283
0.305000	0.003432
0.310000	0.003580
0.315000	0.003726
0.320000	0.003870
0.325000	0.004013
0.330000	0.004154
0.334999	0.004293
0.339999	0.004431
0.344999	0.004566
0.349999	0.004700
0.354999	0.004832
0.359999	0.004963
0.364999	0.005091
0.369999	0.005218
0.374999	0.005343
0.379999	0.005466
0.384999	0.005588
0.389999	0.005708
0.394999	0.005826
0.399999	0.005943
0.404999	0.006058
0.409999	0.006171
0.414999	0.006283
0.419999	0.006393
0.424999	0.006502
0.429999	0.006608
0.434999	0.006714
0.439999	0.006818
0.444999	0.006920

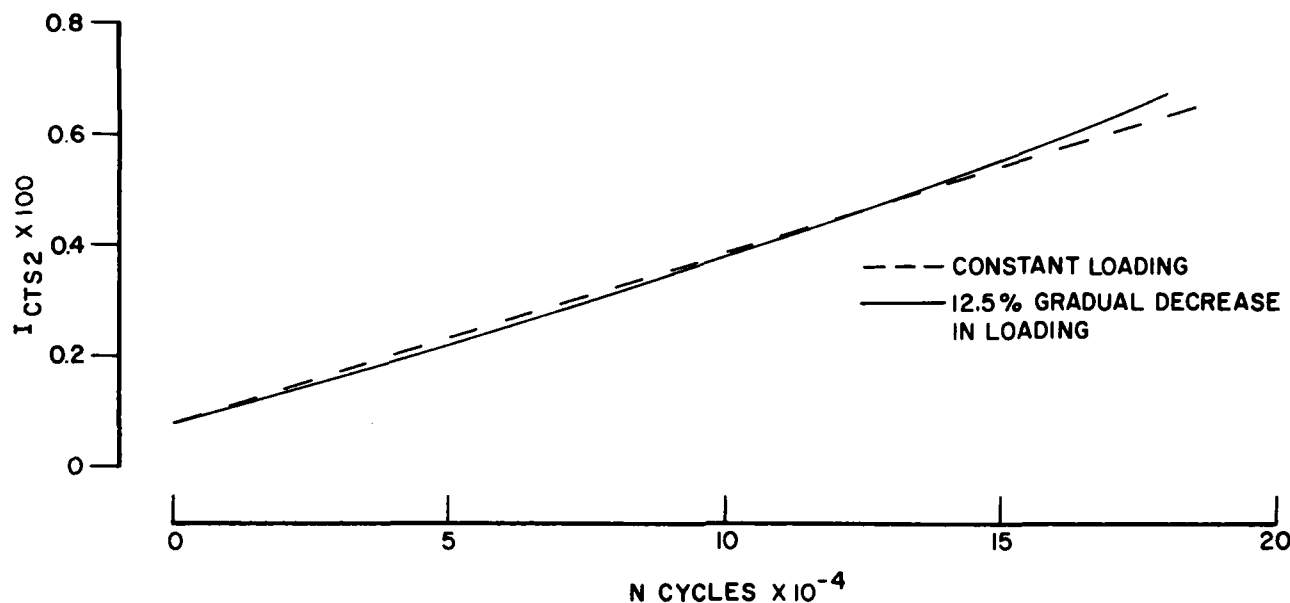


Figure 19. Theoretical comparison of second-power crack growth curves corresponding to a constant range of loading and the gradual decrease in range of loading  $\Delta P = \Delta P_{\text{mean}} (-0.6 a/w + 1.198)$ , of the Lal-Weiss tests.

#### FOURTH-POWER VS SECOND-POWER CTS INTEGRAL REPRESENTATION

The fact that the CTS integral vs  $N$  forms straight line segments shows that the growth rate does increase as the square of the crack tip stress intensity range,  $\Delta K$ . As further support of this, a CTS integral was made up for rate of growth per cycle proportional to the fourth power of  $\Delta K$ , i.e.,  $m = 4$ , which has been advocated by some authors and is not far from  $m \cong 3.7$  given the Lal-Weiss analysis.

From Equations (2) and (25) it is

$$I_{\text{CTS4}} \equiv \int_{x_0}^x \frac{dx}{x^2 (29.6 - 185.5x + 655.7x^2 - 1017x^3 + 638.9x^4)^4} \quad (31)$$

Values of this fourth-power CTS integral were computed and tabulated in condensed form in Table 6. The results, using this growth law, are shown in Figure 20. Here a straight line was drawn to represent the second-power CTS integral vs cycles of growth. At selected numbers of cycles the CTS integral values were read off this second-power line. The second-power CTS integral tables were consulted to find corresponding values of  $a/w$  and these values were then used to find values of the fourth-power CTS integral from the fourth-power integral table.

Table 6. CTS INTEGRAL  $I_{CTS4}$  BASED ON ASTM STANDARD E399-70T FORMULA FOR  
K.  $X \equiv a/w$ ,  $m = 4$ . [SEE EQUATION (31).] CONDENSED TABLE.

X	$I_{CTS4} \times 10^5$	X	$I_{CTS4} \times 10^5$	X	$I_{CTS4} \times 10^5$
.2000	.0000	.4000	17.2679	.6000	20.0824
.2050	.6843	.4050	17.4375	.6050	20.0966
.2100	1.3625	.4100	17.5987	.6100	20.1098
.2150	2.0334	.4150	17.7519	.6150	20.1220
.2200	2.6959	.4200	17.8975	.6200	20.1331
.2250	3.3491	.4250	18.0358	.6250	20.1434
.2300	3.9919	.4300	18.1671	.6300	20.1528
.2350	4.6234	.4350	18.2917	.6350	20.1615
.2400	5.2427	.4400	18.4099	.6400	20.1694
.2450	5.8490	.4450	18.5221	.6450	20.1766
.2500	6.4416	.4500	18.6284	.6500	20.1832
.2550	7.0199	.4550	18.7291	.6550	20.1892
.2600	7.5834	.4600	18.8245	.6600	20.1947
.2650	8.1315	.4650	18.9148	.6650	20.1997
.2700	8.6640	.4700	19.0002	.6700	20.2043
.2750	9.1806	.4750	19.0810	.6750	20.2084
.2800	9.6810	.4800	19.1574	.6800	20.2121
.2850	10.1651	.4850	19.2295	.6850	20.2155
.2900	10.6329	.4900	19.2976	.6900	20.2186
.2950	11.0843	.4950	19.3619	.6950	20.2213
.3000	11.5196	.5000	19.4224	.7000	20.2238
.3050	11.9367	.5050	19.4795	.7050	20.2261
.3100	12.3419	.5100	19.5332	.7100	20.2281
.3150	12.7295	.5150	19.5837	.7150	20.2299
.3200	13.1016	.5200	19.6312	.7200	20.2316
.3250	13.4587	.5250	19.6759	.7250	20.2330
.3300	13.8010	.5300	19.7177	.7300	20.2344
.3350	14.1283	.5350	19.7570	.7350	20.2356
.3400	14.4428	.5400	19.7938	.7400	20.2366
.3450	14.7431	.5450	19.8282	.7450	20.2376
.3500	15.0302	.5500	19.8603	.7500	20.2384
.3550	15.3045	.5550	19.8904	.7550	20.2392
.3600	15.5664	.5600	19.9184	.7600	20.2399
.3650	15.8164	.5650	19.9446	.7650	20.2405
.3700	16.0550	.5700	19.9689	.7700	20.2410
.3750	16.2824	.5750	19.9915	.7750	20.2415
.3800	16.4992	.5800	20.0126	.7800	20.2419
.3850	16.7058	.5850	20.0321	.7850	20.2423
.3900	16.9025	.5900	20.0501	.7900	20.2426
.3950	17.0897	.5950	20.0669	.7950	20.2430
.4000	17.2679	.6000	20.0824	.8000	20.2432

These fourth-power CTS integral values were then plotted on the same graph (Figure 20) as the second-power growth line but, to make the initial slopes of the two curves the same for comparison, unit distance along the fourth-power CTS integral scale was made 26.98 times as long as on the second-power scale. This constant is of course the ratio of the integrands (i.e.,  $dI/dx$ ) of  $I_{CTS2}$  and  $I_{CTS4}$  at the lower limit of these integrals,  $x = 0.2$ .

It is obvious from Figure 20 that data plotted using a fourth-power CTS integral when the crack was actually growing as a second power would form a curve with its concave face pointing down, instead of plotting as a straight line as it would if the data followed a fourth power law.

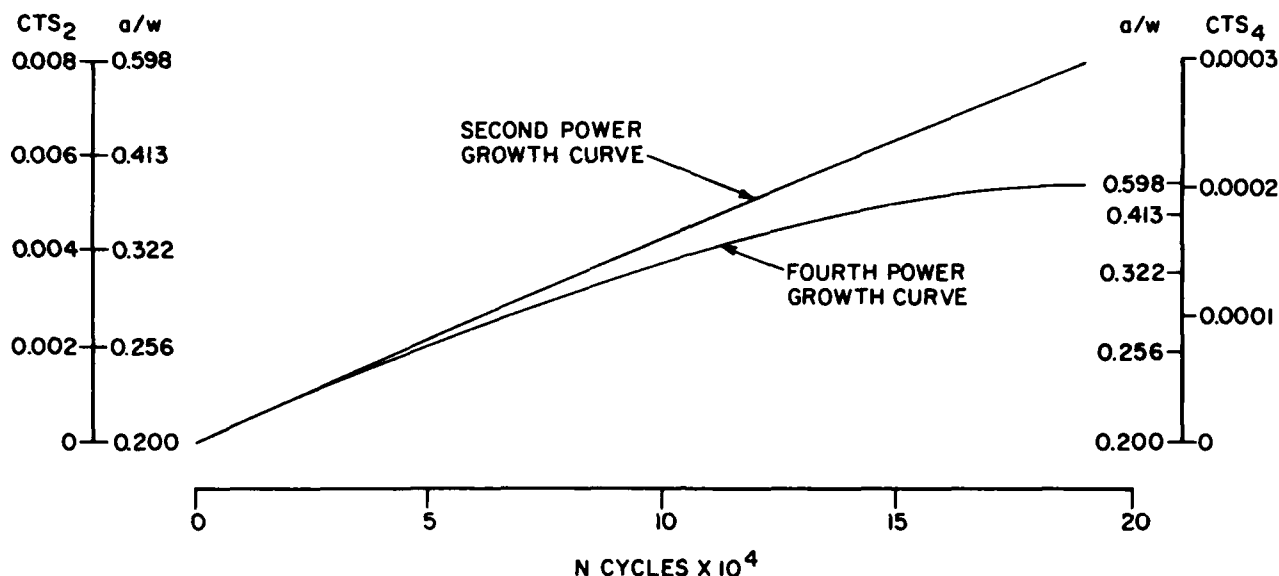


Figure 20. Theoretical comparison of second- and fourth-power crack growth curves.

For the reverse case, using a second-power CTS integral when the crack was actually growing by a fourth power the data would form a curve with its concave face pointing up. It is only in the case where the CTS integral is of the correct power that the CTS integral vs  $N$  curve has a straight line relationship, and on this basis, the correct power, according to the plots of the experimental data, appears to be two.

#### EFFECT OF USING OTHER FORMULAS FOR $K$ THAN THE ASTM STANDARD E399-70T POWER SERIES LAW

Other formulae than that in ASTM Standard E399-70T for computing  $K$  for the CTS have been proposed, such as those in Table 7. To investigate their effect on the matching of the crack growth law, second-power CTS integrals were made using these other relationships for  $\Delta K$  (Appendix A). A straight growth line was drawn to represent a hypothetical CTS integral vs  $N$  line resulting from using the ASTM Standard equation for  $\Delta K$  in the second-power CTS integral. At selected numbers of cycles the ASTM Standard CTS integral values were read off of the line. The CTS integral tables were consulted to get  $a/w$  values, which were then used to find the CTS integral values that were computed as a function of  $a/w$  using the other  $\Delta K$  equations. These CTS values plotted against corresponding values of  $N$  show how data giving a straight line according to the Standard formula, would look if the substitute formulae for  $K$  were used instead.

The resultant plot is shown in Figure 21. The CTS integral scales for each growth curve have been displaced for clarity. From the plot, one can see that there is little difference, aside from a slight curvature, between the shape of the growth curves that resulted from using different equations for computing  $\Delta K$  in the second-power CTS integral. If the scatter in an actual experimental set of data were taken into account, the differences might be considered so negligible that the data could

Table 7. FOUR SECOND-ORDER PADE APPROXIMANT CTS FORMULAE FOR K vs  $a/w \equiv x$  REPRESENTING AND COMPARED WITH ASTM STANDARD E399-70T POWER SERIES, TABLES OF BOWIE AND SRAWLEY AND THE FORMULAE OF BEEUWKES

$x \equiv a/w$	$Kw^{1/2} B/P$ vs $a/w$							
	ASTM E399-70T	Pade	Srawley (H/w) = 0.556	Pade	Bowie	Pade	Beeuwkes	Pade
0.2	5.19	5.19	4.71	4.71	4.52	4.52	4.35	4.35
0.3	5.85	5.99	6.03	5.92	5.48	5.53	5.57	5.56
0.4	7.32	7.32	7.57	7.57	7.08	7.08	7.24	7.24
0.5	9.60	9.59	9.86	9.95	9.56	9.56	9.71	9.73
0.6	13.54	13.54	13.72	13.72	13.70	13.77	13.73	13.73
0.7	21.43	21.00	21.58	20.77	21.59	21.59	21.24	21.16
0.8	37.42	37.42	41.01	41.03	---	(38.77)	39.03	39.03

Standard

$$K = \frac{P}{B w^{1/2}} \left[ \frac{4.5170 - 4.8275x + 7.0528x^2}{1 - 1.3867x + 0.3867x^2} \right]$$

Srawley

$$K = \frac{P}{B w^{1/2}} \left[ \frac{3.0532 - 0.0167x - 3.7528x^2}{1 - 2.1528x + 1.1528x^2} \right]$$

Bowie

$$K = \frac{P}{B w^{1/2}} \left[ \frac{3.4152 - 1.6577x + 3.6408x^2}{1 - 1.5736x + 0.5376x^2} \right]$$

Beeuwkes

$$K = \frac{P}{B w^{1/2}} \left[ \frac{2.7168 + 0.8872x - 3.0716x^2}{1 - 2.0161x + 1.0161x^2} \right]$$

still be taken to correspond to a straight line whichever formulae might be used. In other words, the different formula proposed for computing  $\Delta K$  may not correspond to materially different growth curves in the range of  $a/w$  considered, insofar as comparison with experimental data scatter or variability is concerned. They do suggest however, that in our analysis of a crack growth experiment, the apparent occurrence of a locus of crack growth made up of lines having very small slope difference should be disregarded in favor of a single straight line since a general law of growth must be inferred from, or compared with, nonideal experimental data.

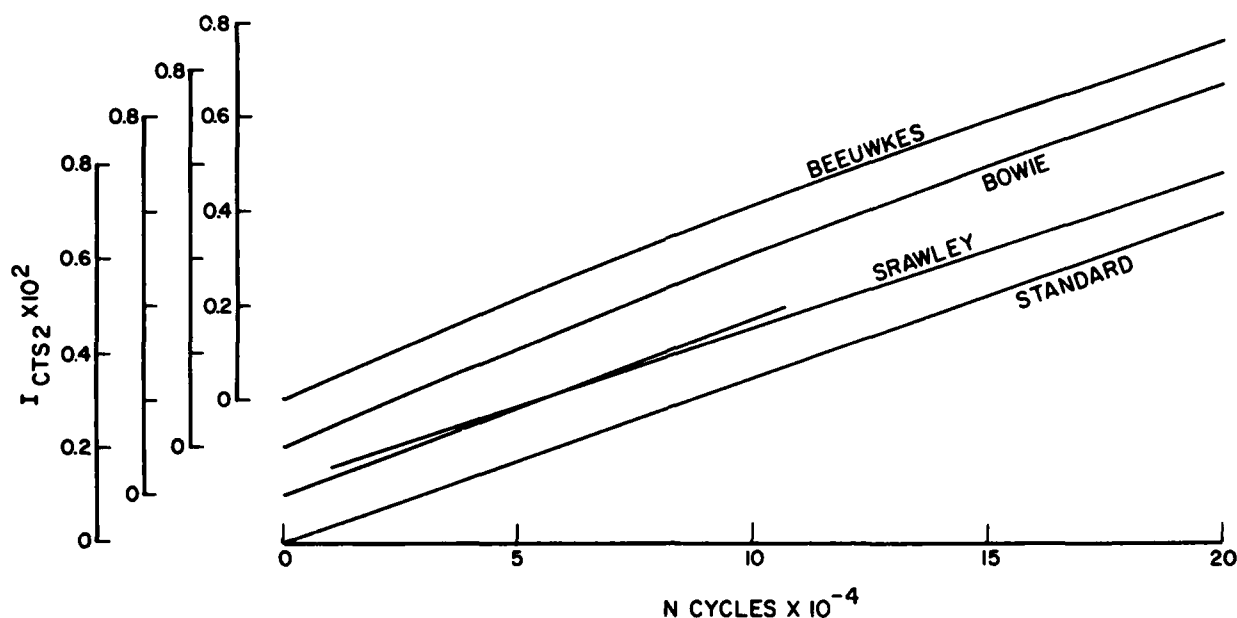


Figure 21. Theoretical comparison of second-power crack growth curves derived from stress intensity  $K$  formulae for Beeuwkes, Bowie, Srawley, and ASTM Standard E399-70T.

#### APPARENT DISCREPANCY $m = 2$ vs $m \cong 3.7$ RESOLVED; $da/dN$ vs $\Delta K$ PLOTS

Our analysis has demonstrated that the crack growth rate of Trip Steels can be completely represented by second-power relations despite the fact that Lal-Weiss, using accepted experimental procedures, obtained a markedly higher power,  $m \cong 3.7$ .

The explanation of this may be found in  $\log da/dN$  vs  $\log \Delta K$  plots, Figures 22, 23, and 24, of all the data on a material, in which the Lal-Weiss points, that correspond to grown crack lengths of only 0.2 in., are encircled. In these plots it is seen that the growth rates in all experiments were proportional to  $(\Delta K)^2$ , even though higher growth rate curves corresponded to larger loads. The Lal-Weiss data points as a group do not fit any experimental growth curve; the points jump from one curve to another but, since all correspond to a fixed crack length, their locus does have the same slope as the envelope of all data points. Except for Specimen B<sub>1</sub> #1, Heat 2321, Type I, practically all data points lie within an envelope consisting of two parallel lines having a slope of four and separated from each other in accordance with PRESENT TYPE OF ANALYSIS.

The  $\Delta K$  and  $da/dN$  values, Table 8, on these figures were computed from crack growths separated by 0.05-in. increments taken from smooth curves drawn through the experimental data as discussed following Equation (28) and shown on the sample data sheets from Lal-Weiss (Figure 2). They were computed for  $m = 2$ , as follows:

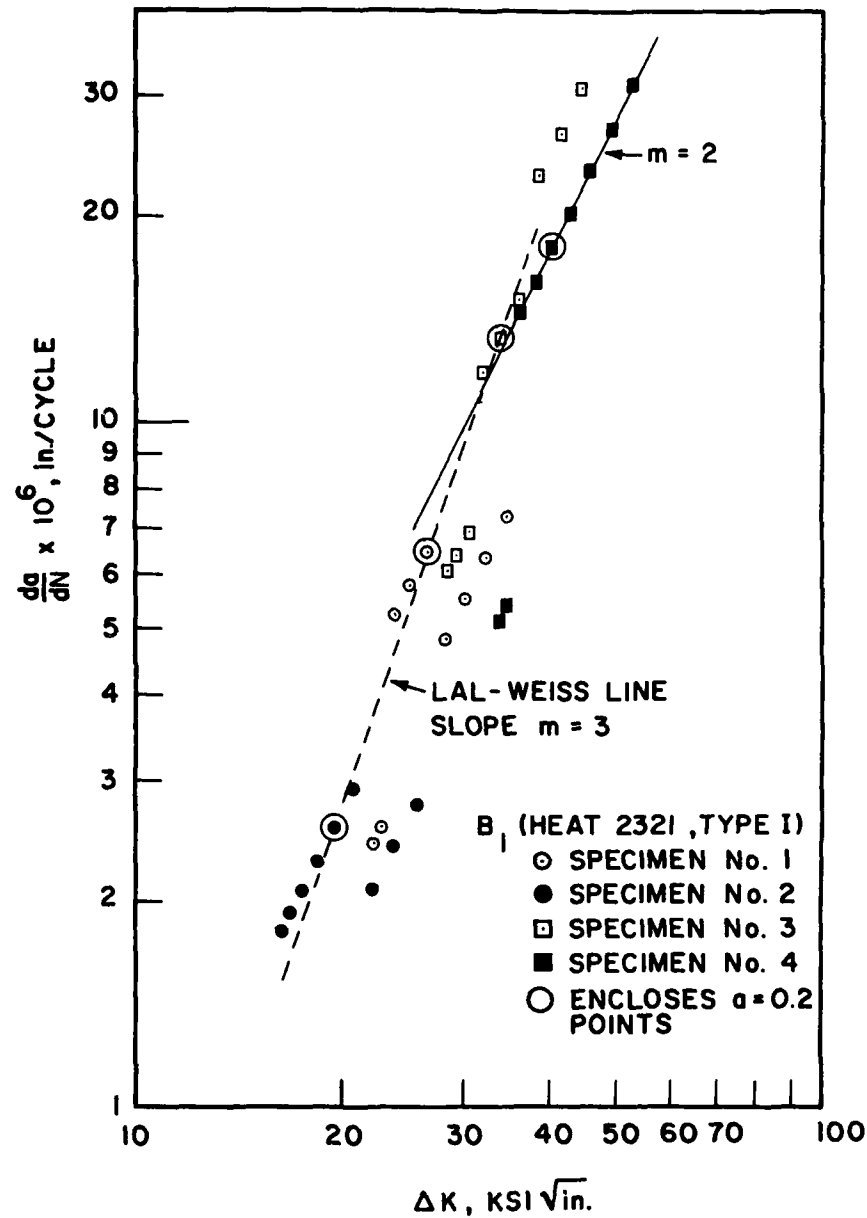


Figure 22. Comparison of representative  $da/dN$  vs  $\Delta K$  data from the individual tests with data based only on grown crack lengths  $a = 0.2$  in., as in Lal-Weiss. All values are from present work. For  $a = 0.2$ -in.,  $da/dN \cong$  Lal-Weiss data but  $\Delta K \cong 1.255 \times$  Lal-Weiss data. Material B<sub>1</sub> (heat 2321, type I).



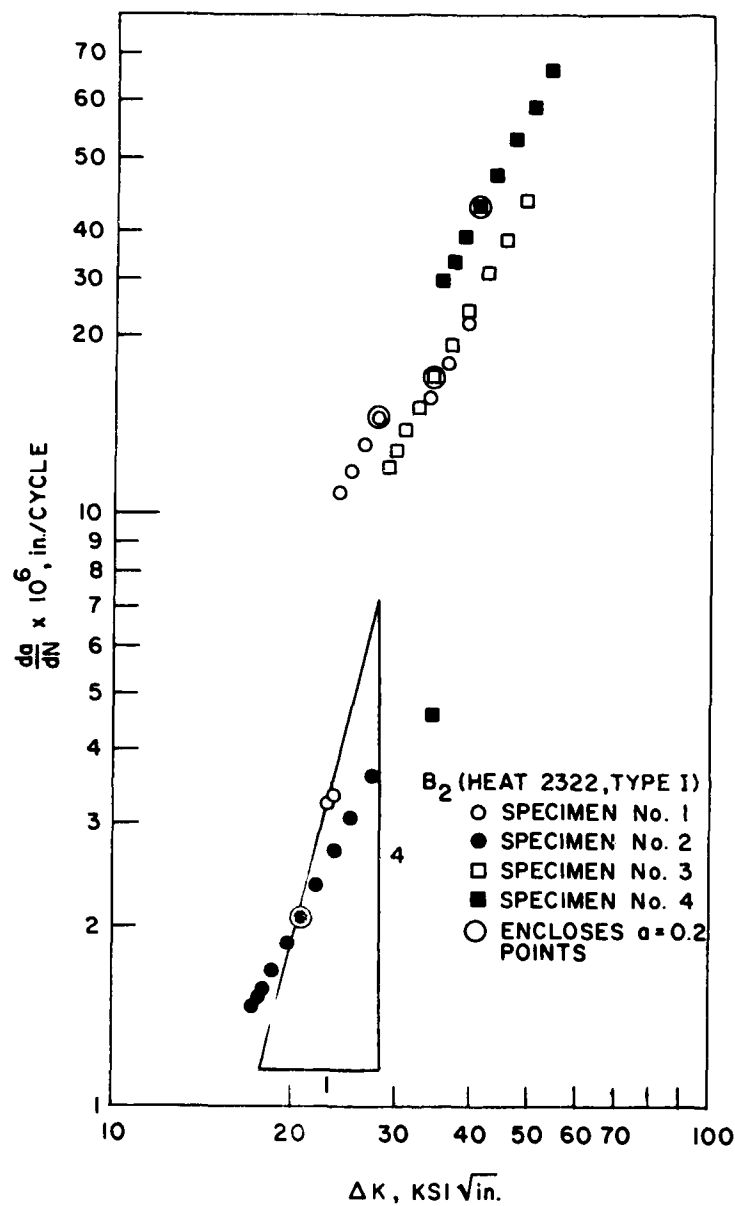


Figure 23. Comparison of representative  $da/dN$  vs  $\Delta K$  data from the individual tests with data based only on grown crack lengths for  $a = 0.2$ -in., as in Lal-Weiss. All values are from present work. For  $a = 0.2$  in.,  $da/dN \cong$  Lal-Weiss data but  $\Delta K \cong 1.255 \times$  Lal-Weiss data. Material B<sub>2</sub> (heat 2322, type I).

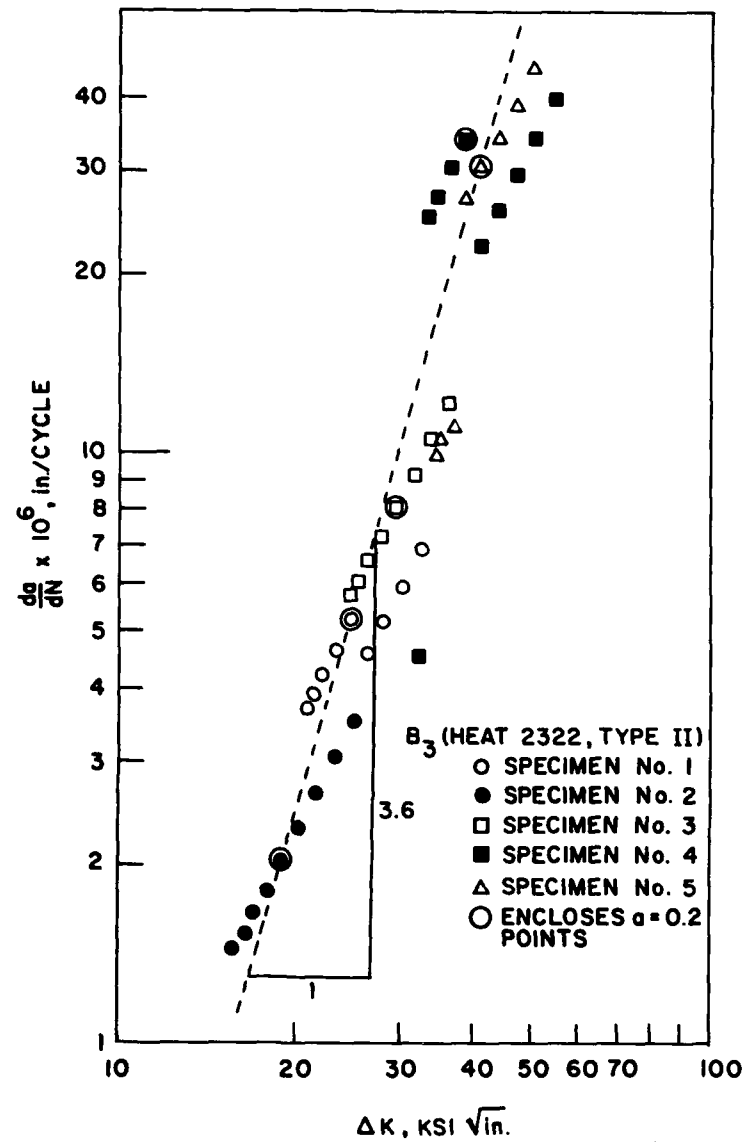


Figure 24. Comparison of representative  $da/dN$  vs  $\Delta K$  data from the individual tests with data based only on grown crack lengths of  $a = 0.2 \text{ in.}$ , as in Lal-Weiss. All values are from present work. For  $a = 0.2 \text{ in.}$ ,  $da/dN \cong \text{Lal-Weiss data}$  but  $\Delta K \cong 1.255 \times \text{Lal-Weiss data}$ . Material B<sub>3</sub> (heat 2322, type II).

Table 8. da/dN AND ΔK FOR FIGURES 22, 23, AND 24

Thk	a, in.	Specimen	Slope x10 <sup>8</sup>	$\frac{da}{dN} \times 10^6$ in./cy.	$\frac{\Delta K}{\sqrt{\text{in.}}}$ ksi	Specimen	Slope x10 <sup>8</sup>	$\frac{da}{dN} \times 10^6$ in./cy.	$\frac{\Delta K}{\sqrt{\text{in.}}}$ ksi
B = 0.300 in.	0.40	B <sub>1I</sub> - 1 ΔP = 1710	4.88	2.43	22.3	B <sub>1I</sub> - 2 ΔP = 1260	3.66	1.82	16.5
	0.425			2.49	22.6			1.87	16.7
	0.45			2.57	23			1.93	16.9
	0.50		9.24	5.26	23.9			2.08	17.6
	0.55			5.81	25.1			2.30	18.5
	0.60			6.52	26.6			2.58	19.6
	0.65		6.07	4.85	28.3		2.29	2.93	20.9
	0.70			5.55	30.3			2.09	22.3
	0.75			6.38	32.5			2.41	23.9
	0.80			7.37	34.9			2.78	25.7
B = 0.285 in.	0.40	B <sub>2I</sub> - 1 ΔP = 1685	6.59	3.28	23.2	B <sub>2I</sub> - 2 ΔP = 1260	2.94	1.46	17.3
	0.425			3.36	23.5			1.50	17.5
	0.45		20.8	10.9	23.8			1.55	17.8
	0.50			11.9	24.8			1.67	18.5
	0.55			13.1	26.1			1.85	19.5
	0.60			14.7	27.6			2.07	20.6
	0.65		15	12.0	29.4			2.35	22
	0.70			13.7	31.4			2.69	23.5
	0.75			15.8	33.7			3.09	25.2
	0.80			18.2	36.2			3.57	27.1
B = 0.315 in.	0.40	B <sub>3II</sub> - 1 ΔP = 1679.7	7.35	3.66	20.9	B <sub>3II</sub> - 2 ΔP = 1294.7	2.88	1.43	15.7
	0.425			3.75	21.2			1.47	16.3
	0.45			3.87	21.5			1.51	16.6
	0.50			4.19	22.4			1.64	17.2
	0.55			4.62	23.5			1.81	18.1
	0.60		5.64	5.19	24.9			2.03	19.2
	0.65			4.51	26.5			2.30	20.4
	0.70			5.15	28.3			2.63	21.8
	0.75			5.92	30.4			3.03	23.4
	0.80			6.85	32.7			3.50	25.2

For B<sub>3II</sub>-2 and B<sub>3II</sub>-3 Lal-Weiss GCL vs N curve was slightly modified to make I<sub>CTS2</sub> vs N a straight line.

$$\Delta K = (\Delta P/B w^{1/2})(x^{1/2})(29.6 - 185.5x + 655.7x^2 - 1017x^3 + 638.9x^4) \text{ from ASTM E399-70T}$$

$$x \equiv a/w \text{ (Figure 1)}$$

$$\frac{da}{dN} = w \left( \frac{\Delta K}{\Delta P/B w^{1/2}} \right)^2 \times \text{Slope where Slope} = (\Delta I_{CTS2}/\Delta N) \text{ on Lal-Weiss Smooth Curves of}$$

Grown Crack Length (GLC) vs Cycles (N) data

Table 8 (Cont). da/dN AND ΔK FOR FIGURES 22, 23, AND 24

Thk	a, in.	Specimen	Slope x10 <sup>8</sup>	$\frac{da}{dN} \times 10^6$ in./cy.	$\Delta K$ ksi $\sqrt{\text{in.}}$	Specimen	Slope x10 <sup>8</sup>	$\frac{da}{dN} \times 10^6$ in./cy.	$\Delta K$ ksi $\sqrt{\text{in.}}$
B = 0.300 in.	0.40	B <sub>1</sub> I - 3 ΔP = 2192		6.07	28.6	B <sub>1</sub> I - 4 ΔP = 2602	10.3	5.13	34
	0.425		12.2	6.22	29			5.25	34.4
	0.45			6.42	29.4			5.42	35
	0.50			6.95	30.6		25.6	14.6	36.4
	0.55		18.9	11.9	32.2			16.1	38.2
	0.60			13.3	34.1			18.1	40.5
	0.65		25.3	15.1	36.3			20.5	43.1
	0.70			23.1	38.8			23.4	46.1
	0.75			26.6	41.6			26.9	49.4
	0.80			30.7	44.7			31.1	53.1
B = 0.285 in.	0.40	B <sub>2</sub> I - 3 ΔP = 2080		12.0	28.6	B <sub>2</sub> I - 4 ΔP = 2477	9.28	4.62	34.1
	0.425		24.1	12.3	29			4.73	34.5
	0.45			12.7	29.4		47.2	24.8	35
	0.50			13.7	30.6			26.9	36.4
	0.55			15.2	32.2			29.7	38.3
	0.60			17	34.1			33.3	40.6
	0.65			19.3	36.3			37.8	43.2
	0.70			22	38.8			43.1	46.2
	0.75			25.3	41.6			49.6	49.5
	0.80			29.3	44.7			57.3	53.2
B = 0.315 in.	0.40	B <sub>3</sub> II - 3 ΔP = 1987.2	11.5	5.72	24.7	B <sub>3</sub> II - 4 ΔP = 2589.5	9.03	4.5	32.2
	0.425			5.87	25			4.61	32.6
	0.45			6.05	24.4		47.6	25	33.1
	0.50			6.55	26.5			27.1	34.5
	0.55			7.23	27.8			29.9	36.2
	0.60			8.11	29.4			33.6	38.4
	0.65			9.20	31.3		28.1	22.5	40.8
	0.70			10.5	33.5			25.7	43.7
	0.75			12.1	35.9			29.5	46.8
	0.80			---	---			34.1	50.3

For B<sub>3</sub>II-2 and B<sub>3</sub>II-3 Lal-Weiss GCL vs N curve was slightly modified to make I<sub>CTS2</sub> vs N a straight line.

$$\Delta K = (\Delta P/B w^{1/2})(x^{1/2})(29.6 - 185.5x + 655.7x^2 - 1017x^3 + 638.9x^4) \text{ from ASTM E399-70T}$$

$x \equiv a/w$  (Figure 1)

$$\frac{da}{dN} = w \left( \frac{\Delta K}{\Delta P/B w^{1/2}} \right)^2 \times \text{Slope where Slope} = (\Delta I_{CTS2}/\Delta N) \text{ on Lal-Weiss Smooth Curves of}$$

Grown Crack Length (GLC) vs Cycles (N) data

Table 8 (Cont).  $da/dN$  AND  $\Delta K$  FOR FIGURES 22, 23, AND 24

Thk	a, in.	Specimen	Slope $\times 10^6$	$\frac{da}{dN} \times 10^6$ in./cy.	$\Delta K$ ksi $\sqrt{\text{in.}}$
B = 0.300 in.	0.40				
	0.425				
	0.45				
	0.50				
	0.55				
	0.60				
	0.65				
	0.70				
	0.75				
	0.80				
B = 0.285 in.	0.40				
	0.425				
	0.45				
	0.50				
	0.55				
	0.60				
	0.65				
	0.70				
	0.75				
	0.80				
B = 0.315 in.	0.40	B <sub>3</sub> II - 5 $\Delta P = 2758$	19.6	9.76	34.3
	0.425			10	34.7
	0.45			10.3	35.3
	0.50			11.2	36.7
	0.55		42.5	26.7	38.6
	0.60			30	40.9
	0.65			34	43.5
	0.70			38.8	46.5
	0.75			44.6	49.9
	0.80			--	--

For B<sub>3</sub>II-2 and B<sub>3</sub>II-3 Lal-Weiss GCL vs N curve was slightly modified to make

$I_{CTS2}$  vs N a straight line.

$\Delta K = (\Delta P/B w^{1/2})(x^{1/2})(29.6 - 185.5x + 655.7x^2 - 1017x^3 + 638.9x^4)$  from ASTM E399-70T

$x \equiv a/w$  (Figure 1)

$\frac{da}{dN} = w \left( \frac{\Delta K}{\Delta P/B w^{1/2}} \right)^2 \times \text{Slope}$  where Slope =  $(\Delta I_{CTS2}/\Delta N)$  on Lal-Weiss Smooth Curves of

Grown Crack Length (GLC) vs Cycles (N) data

$$\frac{da}{dN} = \frac{1}{b^{m/2-1}} \left( \frac{\Delta K}{\Delta S_n} \right)^m (\text{Slope}) \quad , \quad (6)$$

where

$$\text{Slope} = \frac{\int_{a_o}^a \frac{1}{b^{m/2-1}} \left( \frac{\Delta K}{\Delta S_n} \right)^m dN}{N - N_o} = \frac{I_{gm}|_a - I_{gm}|_{a_o}}{N - N_o} \quad (5a)$$

i.e.,

$$\frac{da}{dN} = \left( \frac{\Delta K}{\Delta S_n} \right)^2 \frac{I_{CTS2}|_a - I_{CTS2}|_{a_o}}{N - N_o} \quad , \quad (6)$$

where  $|_a$  means at  $a$ .

The  $da/dN$  values so computed for  $a = 0.4 + 0.2$  in. (i.e., grown crack length = 0.2 in.) were in substantial agreement with the values obtained by Lal-Weiss so that no graphical distinction between the sources of  $da/dN$  data is called for.

However, our  $\Delta K$  values which are computed from ASTM Standard E399-70T, as are the Lal-Weiss values, are about 1.25 times as great as the Lal-Weiss values. Their  $f(a/w) \cong 5$  in  $\Delta K = (\Delta P/B\sqrt{w})f(a/w)$  of this standard according to Figure 2(a), whereas  $f(a/w) = 6.26$  actually, for  $a/w = (0.4 + 0.2)/1.8$  where 0.4 is the initial crack length to which the "grown" crack length of 0.2 in. is to be added to get the total distance from the action line of the pins of the loading fixture to the crack front. Thus, our values of  $\Delta K$  were used in the plots (Figures 22, 23, and 24). (Use of the Lal-Weiss  $\Delta K$  values would simply shift all points to the left a constant amount - an inconsequential difference as far as the Lal-Weiss determination of  $m$  is concerned.)

## CONCLUSIONS

Because of discontinuities in  $da/dN$  that are not readily observable on crack growth curves drawn, as is customary, to idealize periodic observations, it is best to match the actual undifferentiated data with a function (possibly segmented) having plausible characteristics and determine  $da/dN$  from this function.

The crack growth curves of the Trip Steels analyzed here closely follow the Beeuwkes formulation. PRESENT TYPE OF ANALYSIS, which has been used successfully

on many materials and according to which there are discontinuities in  $da/dN$  and except, at these,  $da/dN$  is proportional to  $(\Delta K)^2$ . This agreement follows from the experimental results whether or not there may have been nonuniform material or a crack front that changes shape. Appendix A is supplied herewith to facilitate plotting growth curves by this formulation.

The apparent discrepancy between the Lal-Weiss relation  $da/dN \sim (\Delta K)^{3.7}$  and our  $da/dN \sim \text{Const.} \times (\Delta K)^2$  is that their relation does not correspond to a rate of growth along an actual growth curve while ours does.  $da/dN, \Delta K$  points that Lal and Weiss plotted to get their relation, jump from a point on one growth curve to a corresponding point on another, and so on, and the change in rate for a given range of  $\Delta K$ , along such a relation is not the same as the change on a single growth curve. Their  $m = \sim 3.7$  ( $\sim 4$ ) is the slope of a line connecting corresponding points on the different segments (all having a slope of  $m = 2$ ) of the  $\log da/dN$  vs  $\log \Delta K$  diagram and is parallel to the envelopes connecting line segment ends in this diagram since all the segments seem to have the same length.

Although there is no growth curve for which  $da/dN$  is proportional to  $(\Delta K)^4$ , one may argue (in conformance with the Beeuwkes formulation) that in a certain statistical sense the fourth power law may be considered correct. Namely, for the average performance of a large number of samples of a material or in the case that the growth or increase in  $\Delta K$  to double  $\Delta K$  renders  $\Delta K$  still so small as to be inconsequential for some application; under constant stress this means that the growth in a crack length to double that length makes the length still too small to be considered important. However, even if the length is not negligible, there is a fourth-power envelope line to the growth curves that may confidently be arbitrarily assumed for conservative design to be the growth curve. (See PRESENT TYPE OF ANALYSIS and SECOND-POWER CTS INTEGRAL AND ANALYSIS;  $\Delta K$  BY ASTM STANDARD E399-70T.)

#### ACKNOWLEDGEMENT

Anna Hansen of AMMRC devised the computer programs for the Integral Tables and Robert Tremblay and Leonard Carlson, also of AMMRC, assisted in the analysis of the Trip Steel data.

## APPENDIX A. SECOND-POWER CTS INTEGRALS

CTS Integrals,  $I_{CTS2}$  vs  $x \equiv a/w$ , for Constant Loading Range for ASTM Standard E399-70T Power Series for K, Second-Degree Pade Formulae for Beeuwkes and Bowie K and Third-Degree Pade Formula for Srawley K, follow in Table A-1.



Table A-1

X	STANDARDS	RAWLEY	BOWIE	BEEUWKES	X	STANDARDS	RAWLEY	BOWIE	BEEUWKES
.2000	.000000	.000000	.000000	.000000	.2250	.000915	.001060	.001168	.001246
.2005	.000019	.000023	.000024	.000026	.2255	.000933	.001080	.001190	.001269
.2010	.000037	.000045	.000049	.000053	.2260	.000951	.001100	.001212	.001293
.2015	.000056	.000067	.000073	.000079	.2265	.000969	.001119	.001235	.001316
.2020	.000074	.000090	.000097	.000105	.2270	.000987	.001139	.001257	.001339
.2025	.000093	.000112	.000122	.000131	.2275	.001005	.001159	.001279	.001362
.2030	.000111	.000134	.000146	.000158	.2280	.001023	.001179	.001301	.001386
.2035	.000130	.000156	.000170	.000184	.2285	.001041	.001198	.001323	.001409
.2040	.000148	.000178	.000194	.000210	.2290	.001059	.001218	.001345	.001432
.2045	.000167	.000201	.000218	.000235	.2295	.001076	.001237	.001367	.001455
.2050	.000185	.000223	.000242	.000261	.2300	.001094	.001257	.001389	.001478
.2055	.000203	.000244	.000266	.000287	.2305	.001112	.001276	.001411	.001500
.2060	.000222	.000266	.000290	.000313	.2310	.001130	.001295	.001432	.001523
.2065	.000240	.000288	.000314	.000339	.2315	.001148	.001315	.001454	.001546
.2070	.000259	.000310	.000338	.000364	.2320	.001166	.001334	.001476	.001569
.2075	.000277	.000332	.000362	.000390	.2325	.001183	.001353	.001498	.001591
.2080	.000296	.000353	.000385	.000415	.2330	.001201	.001373	.001519	.001614
.2085	.000314	.000375	.000409	.000441	.2335	.001219	.001392	.001541	.001636
.2090	.000332	.000397	.000433	.000466	.2340	.001237	.001411	.001562	.001659
.2095	.000351	.000418	.000457	.000491	.2345	.001254	.001430	.001584	.001681
.2100	.000369	.000440	.000480	.000516	.2350	.001272	.001449	.001605	.001704
.2105	.000387	.000461	.000504	.000542	.2355	.001290	.001468	.001627	.001726
.2110	.000406	.000482	.000527	.000567	.2360	.001307	.001487	.001648	.001748
.2115	.000424	.000504	.000551	.000592	.2365	.001325	.001506	.001670	.001770
.2120	.000443	.000525	.000574	.000617	.2370	.001343	.001525	.001691	.001792
.2125	.000461	.000546	.000597	.000642	.2375	.001360	.001543	.001712	.001815
.2130	.000479	.000567	.000621	.000667	.2380	.001378	.001562	.001733	.001837
.2135	.000497	.000588	.000644	.000691	.2385	.001395	.001581	.001755	.001859
.2140	.000516	.000609	.000667	.000716	.2390	.001413	.001600	.001776	.001880
.2145	.000534	.000630	.000691	.000741	.2395	.001430	.001618	.001797	.001902
.2150	.000552	.000651	.000714	.000766	.2400	.001448	.001637	.001818	.001924
.2155	.000571	.000672	.000737	.000790	.2405	.001465	.001655	.001839	.001946
.2160	.000589	.000693	.000760	.000815	.2410	.001483	.001674	.001860	.001968
.2165	.000607	.000714	.000783	.000839	.2415	.001500	.001692	.001881	.001989
.2170	.000625	.000735	.000806	.000863	.2420	.001518	.001711	.001902	.002011
.2175	.000643	.000755	.000829	.000888	.2425	.001535	.001729	.001923	.002032
.2180	.000662	.000776	.000852	.000912	.2430	.001553	.001747	.001943	.002054
.2185	.000680	.000797	.000875	.000936	.2435	.001570	.001765	.001964	.002075
.2190	.000698	.000817	.000897	.000961	.2440	.001587	.001784	.001985	.002097
.2195	.000716	.000838	.000920	.000985	.2445	.001605	.001802	.002006	.002118
.2200	.000734	.000858	.000943	.001009	.2450	.001622	.001820	.002026	.002139
.2205	.000752	.000879	.000966	.001033	.2455	.001639	.001838	.002047	.002161
.2210	.000771	.000899	.000988	.001057	.2460	.001657	.001856	.002067	.002182
.2215	.000789	.000919	.001011	.001080	.2465	.001674	.001874	.002088	.002203
.2220	.000807	.000939	.001033	.001104	.2470	.001691	.001892	.002108	.002224
.2225	.000825	.000960	.001056	.001128	.2475	.001708	.001910	.002129	.002245
.2230	.000843	.000980	.001078	.001152	.2480	.001726	.001928	.002149	.002266
.2235	.000861	.001000	.001101	.001175	.2485	.001743	.001946	.002170	.002287
.2240	.000879	.001020	.001123	.001199	.2490	.001760	.001963	.002190	.002308
.2245	.000897	.001040	.001146	.001222	.2495	.001777	.001981	.002210	.002329
.2250	.000915	.001060	.001168	.001246	.2500	.001794	.001999	.002230	.002349

Table A-1 (cont.)

X	STANDARDS	RAWLEY	BOWIE	BEEUNKES	X	STANDARDS	RAWLEY	BOWIE	BEEUNKES
.2500	.001794	.001999	.002230	.002349	.2750	.002621	.002833	.003191	.003325
.2505	.001811	.002017	.002251	.002370	.2755	.002637	.002849	.003210	.003344
.2510	.001828	.002034	.002271	.002391	.2760	.002653	.002864	.003228	.003362
.2515	.001845	.002052	.002291	.002411	.2765	.002669	.002880	.003246	.003380
.2520	.001862	.002069	.002311	.002432	.2770	.002685	.002896	.003264	.003398
.2525	.001879	.002087	.002331	.002453	.2775	.002701	.002911	.003282	.003416
.2530	.001896	.002104	.002351	.002473	.2780	.002717	.002927	.003300	.003434
.2535	.001913	.002122	.002371	.002493	.2785	.002732	.002942	.003318	.003452
.2540	.001930	.002139	.002391	.002514	.2790	.002748	.002958	.003336	.003470
.2545	.001947	.002156	.002411	.002534	.2795	.002764	.002973	.003354	.003488
.2550	.001964	.002174	.002431	.002554	.2800	.002780	.002989	.003372	.003506
.2555	.001981	.002191	.002450	.002575	.2805	.002795	.003004	.003390	.003524
.2560	.001998	.002208	.002470	.002595	.2810	.002811	.003019	.003408	.003542
.2565	.002015	.002225	.002490	.002615	.2815	.002827	.003034	.003425	.003560
.2570	.002032	.002242	.002510	.002635	.2820	.002842	.003050	.003443	.003577
.2575	.002048	.002260	.002529	.002655	.2825	.002858	.003065	.003461	.003595
.2580	.002065	.002277	.002549	.002675	.2830	.002873	.003080	.003478	.003613
.2585	.002082	.002294	.002568	.002695	.2835	.002889	.003095	.003496	.003630
.2590	.002099	.002311	.002588	.002715	.2840	.002904	.003110	.003514	.003648
.2595	.002115	.002327	.002607	.002735	.2845	.002920	.003125	.003531	.003665
.2600	.002132	.002344	.002627	.002754	.2850	.002935	.003140	.003549	.003683
.2605	.002149	.002361	.002646	.002774	.2855	.002951	.003155	.003566	.003700
.2610	.002165	.002378	.002665	.002794	.2860	.002966	.003170	.003583	.003718
.2615	.002182	.002395	.002685	.002813	.2865	.002981	.003185	.003601	.003735
.2620	.002199	.002412	.002704	.002833	.2870	.002997	.003200	.003618	.003752
.2625	.002215	.002428	.002723	.002852	.2875	.003012	.003215	.003636	.003769
.2630	.002232	.002445	.002743	.002872	.2880	.003027	.003230	.003653	.003787
.2635	.002248	.002462	.002762	.002891	.2885	.003043	.003245	.003670	.003804
.2640	.002265	.002478	.002781	.002911	.2890	.003058	.003259	.003687	.003821
.2645	.002281	.002495	.002800	.002930	.2895	.003073	.003274	.003704	.003838
.2650	.002298	.002511	.002819	.002949	.2900	.003088	.003289	.003721	.003855
.2655	.002314	.002528	.002838	.002969	.2905	.003103	.003303	.003739	.003872
.2660	.002330	.002544	.002857	.002988	.2910	.003118	.003318	.003756	.003889
.2665	.002347	.002560	.002876	.003007	.2915	.003134	.003333	.003773	.003906
.2670	.002363	.002577	.002895	.003026	.2920	.003149	.003347	.003790	.003923
.2675	.002380	.002593	.002914	.003045	.2925	.003164	.003362	.003807	.003939
.2680	.002396	.002609	.002932	.003064	.2930	.003179	.003376	.003823	.003956
.2685	.002412	.002626	.002951	.003083	.2935	.003194	.003391	.003840	.003973
.2690	.002428	.002642	.002970	.003102	.2940	.003209	.003405	.003857	.003990
.2695	.002445	.002658	.002988	.003121	.2945	.003224	.003419	.003874	.004006
.2700	.002461	.002674	.003007	.003140	.2950	.003239	.003434	.003891	.004023
.2705	.002477	.002690	.003026	.003158	.2955	.003253	.003448	.003907	.004039
.2710	.002493	.002706	.003044	.003177	.2960	.003268	.003462	.003924	.004056
.2715	.002509	.002722	.003063	.003196	.2965	.003283	.003477	.003941	.004072
.2720	.002525	.002738	.003081	.003214	.2970	.003298	.003491	.003957	.004089
.2725	.002541	.002754	.003100	.003233	.2975	.003313	.003505	.003974	.004105
.2730	.002558	.002770	.003118	.003252	.2980	.003327	.003519	.003990	.004122
.2735	.002574	.002786	.003137	.003270	.2985	.003342	.003533	.004007	.004138
.2740	.002590	.002802	.003155	.003288	.2990	.003357	.003547	.004023	.004154
.2745	.002606	.002817	.003173	.003307	.2995	.003371	.003561	.004040	.004170
.2750	.002621	.002833	.003191	.003325	.3000	.003386	.003575	.004056	.004187

Table A-1 (cont)

X	STANDARDS	SAWLEY	BOWIE	BEEUWKES	X	STANDARDS	SAWLEY	BOWIE	BEEUWKES
3000	.003386	.003575	.004056	.004187	3250	.004082	.004236	.004829	.004945
3005	.003401	.003589	.004072	.004203	3255	.004095	.004249	.004844	.004959
3010	.003415	.003603	.004089	.004219	3260	.004108	.004261	.004859	.004974
3015	.003430	.003617	.004105	.004235	3265	.004121	.004274	.004873	.004988
3020	.003444	.003631	.004121	.004251	3270	.004135	.004286	.004888	.005002
3025	.003459	.003645	.004137	.004267	3275	.004148	.004298	.004902	.005016
3030	.003473	.003659	.004154	.004283	3280	.004161	.004311	.004916	.005030
3035	.003488	.003673	.004170	.004299	3285	.004174	.004323	.004931	.005044
3040	.003502	.003688	.004186	.004315	3290	.004187	.004335	.004945	.005058
3045	.003516	.003700	.004202	.004330	3295	.004200	.004348	.004959	.005072
3050	.003531	.003714	.004218	.004346	3300	.004213	.004360	.004974	.005086
3055	.003545	.003728	.004234	.004362	3305	.004226	.004372	.004988	.005100
3060	.003559	.003741	.004250	.004378	3310	.004239	.004384	.005002	.005113
3065	.003574	.003755	.004266	.004393	3315	.004251	.004396	.005016	.005127
3070	.003588	.003768	.004282	.004409	3320	.004264	.004408	.005030	.005141
3075	.003602	.003782	.004297	.004424	3325	.004277	.004420	.005044	.005155
3080	.003616	.003795	.004313	.004440	3330	.004290	.004432	.005059	.005168
3085	.003630	.003809	.004329	.004455	3335	.004303	.004444	.005073	.005182
3090	.003645	.003822	.004345	.004471	3340	.004315	.004456	.005087	.005195
3095	.003659	.003836	.004360	.004486	3345	.004328	.004468	.005101	.005209
3100	.003673	.003849	.004376	.004502	3350	.004341	.004480	.005114	.005222
3105	.003687	.003862	.004392	.004517	3355	.004353	.004492	.005128	.005236
3110	.003701	.003876	.004407	.004532	3360	.004366	.004504	.005142	.005249
3115	.003715	.003889	.004423	.004548	3365	.004379	.004516	.005156	.005263
3120	.003729	.003902	.004438	.004563	3370	.004391	.004528	.005170	.005276
3125	.003743	.003916	.004454	.004578	3375	.004404	.004539	.005184	.005290
3130	.003757	.003929	.004469	.004593	3380	.004416	.004551	.005197	.005303
3135	.003770	.003942	.004485	.004608	3385	.004429	.004563	.005211	.005316
3140	.003784	.003955	.004500	.004623	3390	.004441	.004574	.005225	.005329
3145	.003798	.003968	.004515	.004638	3395	.004454	.004586	.005238	.005343
3150	.003812	.003981	.004531	.004653	3400	.004466	.004598	.005252	.005356
3155	.003826	.003994	.004546	.004668	3405	.004478	.004609	.005265	.005369
3160	.003839	.004007	.004561	.004683	3410	.004491	.004621	.005279	.005382
3165	.003853	.004020	.004576	.004698	3415	.004503	.004633	.005293	.005395
3170	.003867	.004033	.004592	.004713	3420	.004515	.004644	.005306	.005408
3175	.003880	.004046	.004607	.004728	3425	.004528	.004656	.005319	.005421
3180	.003894	.004059	.004622	.004743	3430	.004540	.004667	.005333	.005434
3185	.003908	.004072	.004637	.004757	3435	.004552	.004679	.005346	.005447
3190	.003921	.004085	.004652	.004772	3440	.004564	.004690	.005360	.005460
3195	.003935	.004098	.004667	.004787	3445	.004576	.004701	.005373	.005473
3200	.003948	.004110	.004682	.004801	3450	.004589	.004713	.005386	.005486
3205	.003962	.004123	.004697	.004816	3455	.004601	.004724	.005399	.005498
3210	.003975	.004136	.004712	.004830	3460	.004613	.004735	.005413	.005511
3215	.003989	.004148	.004726	.004845	3465	.004625	.004747	.005426	.005524
3220	.004002	.004161	.004741	.004859	3470	.004637	.004758	.005439	.005537
3225	.004015	.004174	.004756	.004874	3475	.004649	.004769	.005452	.005549
3230	.004029	.004186	.004771	.004888	3480	.004661	.004780	.005465	.005562
3235	.004042	.004199	.004785	.004902	3485	.004673	.004792	.005478	.005574
3240	.004055	.004211	.004800	.004917	3490	.004685	.004803	.005491	.005587
3245	.004069	.004224	.004815	.004931	3495	.004697	.004814	.005504	.005600
3250	.004082	.004236	.004829	.004945	3500	.004708	.004825	.005517	.005612

Table A-1 (cont)

X	STANDARDS	RAWLEY	BOWIE	BEEUWKES	X	STANDARDS	RAWLEY	BOWIE	BEEUWKES
.3500	.004708	.004825	.005517	.005612	.3750	.005268	.005348	.006125	.006196
.3505	.004720	.004836	.005530	.005624	.3755	.005278	.005358	.006136	.006207
.3510	.004732	.004847	.005543	.005637	.3760	.005289	.005368	.006147	.006218
.3515	.004744	.004858	.005556	.005649	.3765	.005299	.005378	.006159	.006229
.3520	.004756	.004869	.005569	.005662	.3770	.005310	.005388	.006170	.006240
.3525	.004767	.004880	.005581	.005674	.3775	.005320	.005397	.006181	.006251
.3530	.004779	.004891	.005594	.005686	.3780	.005330	.005407	.006193	.006261
.3535	.004791	.004902	.005607	.005699	.3785	.005341	.005417	.006204	.006272
.3540	.004802	.004913	.005620	.005711	.3790	.005351	.005426	.006215	.006283
.3545	.004814	.004924	.005632	.005723	.3795	.005361	.005436	.006226	.006293
.3550	.004825	.004935	.005645	.005735	.3800	.005372	.005446	.006237	.006304
.3555	.004837	.004945	.005657	.005747	.3805	.005382	.005455	.006248	.006315
.3560	.004849	.004956	.005670	.005759	.3810	.005392	.005465	.006259	.006325
.3565	.004860	.004967	.005683	.005772	.3815	.005402	.005475	.006270	.006336
.3570	.004872	.004978	.005695	.005784	.3820	.005413	.005484	.006281	.006346
.3575	.004883	.004989	.005707	.005796	.3825	.005423	.005494	.006292	.006357
.3580	.004894	.004999	.005720	.005808	.3830	.005433	.005503	.006303	.006367
.3585	.004906	.005010	.005732	.005819	.3835	.005443	.005513	.006314	.006378
.3590	.004917	.005021	.005745	.005831	.3840	.005453	.005522	.006325	.006388
.3595	.004929	.005031	.005757	.005843	.3845	.005463	.005531	.006336	.006399
.3600	.004940	.005042	.005769	.005855	.3850	.005473	.005541	.006347	.006409
.3605	.004951	.005052	.005782	.005867	.3855	.005483	.005550	.006358	.006419
.3610	.004963	.005063	.005794	.005879	.3860	.005493	.005560	.006368	.006430
.3615	.004974	.005073	.005806	.005891	.3865	.005503	.005569	.006379	.006440
.3620	.004985	.005084	.005818	.005902	.3870	.005513	.005578	.006390	.006450
.3625	.004996	.005094	.005831	.005914	.3875	.005523	.005588	.006400	.006460
.3630	.005007	.005105	.005843	.005926	.3880	.005533	.005597	.006411	.006471
.3635	.005018	.005115	.005855	.005937	.3885	.005543	.005606	.006422	.006481
.3640	.005030	.005126	.005867	.005949	.3890	.005553	.005615	.006432	.006491
.3645	.005041	.005136	.005879	.005960	.3895	.005563	.005625	.006443	.006501
.3650	.005052	.005146	.005891	.005972	.3900	.005572	.005634	.006453	.006511
.3655	.005063	.005157	.005903	.005983	.3905	.005582	.005643	.006464	.006521
.3660	.005074	.005167	.005915	.005995	.3910	.005592	.005652	.006474	.006531
.3665	.005085	.005177	.005927	.006006	.3915	.005602	.005661	.006485	.006541
.3670	.005096	.005188	.005939	.006018	.3920	.005611	.005670	.006495	.006551
.3675	.005107	.005198	.005950	.006029	.3925	.005621	.005679	.006506	.006561
.3680	.005118	.005208	.005962	.006041	.3930	.005631	.005688	.006516	.006571
.3685	.005128	.005218	.005974	.006052	.3935	.005640	.005697	.006526	.006581
.3690	.005139	.005228	.005986	.006063	.3940	.005650	.005706	.006537	.006591
.3695	.005150	.005238	.005998	.006074	.3945	.005660	.005715	.006547	.006600
.3700	.005161	.005249	.006009	.006086	.3950	.005669	.005724	.006557	.006610
.3705	.005172	.005259	.006021	.006097	.3955	.005679	.005733	.006568	.006620
.3710	.005182	.005269	.006033	.006108	.3960	.005688	.005742	.006578	.006630
.3715	.005193	.005279	.006044	.006119	.3965	.005698	.005751	.006588	.006640
.3720	.005204	.005289	.006056	.006130	.3970	.005707	.005760	.006598	.006649
.3725	.005215	.005299	.006067	.006141	.3975	.005717	.005769	.006608	.006659
.3730	.005225	.005309	.006079	.006152	.3980	.005726	.005778	.006618	.006669
.3735	.005236	.005319	.006090	.006163	.3985	.005736	.005786	.006628	.006678
.3740	.005246	.005329	.006102	.006174	.3990	.005745	.005795	.006638	.006688
.3745	.005257	.005338	.006113	.006185	.3995	.005754	.005804	.006648	.006697
.3750	.005268	.005348	.006125	.006196	.4000	.005764	.005813	.006658	.006707

Table A-1 (cont.)

X	STANDARDS	RAWLEY	BOWIE	BEEUWKES	X	STANDARDS	RAWLEY	BOWIE	BEEUWKES
.4000	.005764	.005813	.006658	.006707	.4250	.006201	.006224	.007124	.007151
.4005	.005773	.005821	.006668	.006716	.4255	.006210	.006231	.007133	.007160
.4010	.005782	.005830	.006678	.006726	.4260	.006218	.006239	.007141	.007168
.4015	.005792	.005839	.006688	.006735	.4265	.006226	.006247	.007150	.007176
.4020	.005801	.005848	.006698	.006745	.4270	.006234	.006254	.007159	.007184
.4025	.005810	.005856	.006708	.006754	.4275	.006242	.006262	.007167	.007192
.4030	.005819	.005865	.006718	.006764	.4280	.006250	.006270	.007176	.007201
.4035	.005828	.005873	.006728	.006773	.4285	.006258	.006277	.007184	.007209
.4040	.005837	.005882	.006737	.006782	.4290	.006266	.006285	.007193	.007217
.4045	.005847	.005891	.006747	.006792	.4295	.006274	.006292	.007201	.007225
.4050	.005856	.005899	.006757	.006801	.4300	.006283	.006300	.007210	.007233
.4055	.005865	.005908	.006767	.006810	.4305	.006290	.006307	.007218	.007241
.4060	.005874	.005916	.006776	.006819	.4310	.006298	.006315	.007226	.007249
.4065	.005883	.005925	.006786	.006828	.4315	.006306	.006322	.007235	.007257
.4070	.005892	.005933	.006795	.006838	.4320	.006314	.006330	.007243	.007265
.4075	.005901	.005941	.006805	.006847	.4325	.006322	.006337	.007251	.007273
.4080	.005910	.005950	.006815	.006856	.4330	.006330	.006345	.007260	.007281
.4085	.005919	.005958	.006824	.006865	.4335	.006338	.006352	.007268	.007289
.4090	.005928	.005967	.006834	.006874	.4340	.006346	.006359	.007276	.007297
.4095	.005937	.005975	.006843	.006883	.4345	.006354	.006367	.007285	.007304
.4100	.005945	.005983	.006853	.006892	.4350	.006361	.006374	.007293	.007312
.4105	.005954	.005992	.006862	.006901	.4355	.006369	.006381	.007301	.007320
.4110	.005963	.006000	.006871	.006910	.4360	.006377	.006389	.007309	.007328
.4115	.005972	.006008	.006881	.006919	.4365	.006385	.006396	.007317	.007336
.4120	.005981	.006016	.006890	.006928	.4370	.006392	.006403	.007325	.007343
.4125	.005990	.006025	.006899	.006937	.4375	.006400	.006411	.007333	.007351
.4130	.005998	.006033	.006906	.006946	.4380	.006408	.006418	.007341	.007359
.4135	.006007	.006041	.006918	.006955	.4385	.006415	.006425	.007349	.007366
.4140	.006016	.006049	.006927	.006963	.4390	.006423	.006432	.007357	.007374
.4145	.006024	.006057	.006936	.006972	.4395	.006431	.006439	.007365	.007382
.4150	.006033	.006065	.006946	.006981	.4400	.006438	.006446	.007373	.007389
.4155	.006042	.006074	.006955	.006990	.4405	.006446	.006454	.007381	.007397
.4160	.006050	.006082	.006964	.006998	.4410	.006453	.006461	.007389	.007404
.4165	.006059	.006090	.006973	.007007	.4415	.006461	.006468	.007397	.007412
.4170	.006067	.006098	.006982	.007016	.4420	.006469	.006475	.007405	.007420
.4175	.006076	.006106	.006991	.007025	.4425	.006476	.006482	.007413	.007427
.4180	.006084	.006114	.007000	.007033	.4430	.006483	.006489	.007421	.007434
.4185	.006093	.006122	.007009	.007042	.4435	.006491	.006496	.007428	.007442
.4190	.006101	.006130	.007018	.007050	.4440	.006498	.006503	.007436	.007449
.4195	.006110	.006138	.007027	.007059	.4445	.006506	.006510	.007444	.007457
.4200	.006118	.006146	.007036	.007067	.4450	.006513	.006517	.007452	.007464
.4205	.006127	.006153	.007045	.007076	.4455	.006521	.006524	.007459	.007472
.4210	.006135	.006161	.007054	.007084	.4460	.006528	.006531	.007467	.007479
.4215	.006143	.006169	.007063	.007093	.4465	.006535	.006538	.007475	.007486
.4220	.006152	.006177	.007072	.007101	.4470	.006543	.006545	.007482	.007493
.4225	.006160	.006185	.007080	.007110	.4475	.006550	.006552	.007490	.007501
.4230	.006168	.006193	.007089	.007118	.4480	.006557	.006558	.007498	.007508
.4235	.006177	.006200	.007098	.007126	.4485	.006564	.006565	.007505	.007515
.4240	.006185	.006208	.007107	.007135	.4490	.006572	.006572	.007513	.007522
.4245	.006193	.006216	.007115	.007143	.4495	.006579	.006579	.007520	.007530
.4250	.006201	.006224	.007124	.007151	.4500	.006586	.006586	.007528	.007537

Table A-1 (cont)

X	STANDARDS	RAWLEY	BOWIE	BEEUWKES	X	STANDARDS	RAWLEY	BOWIE	BEEUWKES
.4500	.006586	.006586	.007528	.007537	.4750	.006922	.006903	.007875	.007870
.4505	.006593	.006593	.007535	.007544	.4755	.006928	.006909	.007882	.007876
.4510	.006600	.006599	.007543	.007551	.4760	.006935	.006915	.007888	.007882
.4515	.006608	.006606	.007550	.007558	.4765	.006941	.006921	.007894	.007888
.4520	.006615	.006613	.007558	.007565	.4770	.006947	.006927	.007901	.007894
.4525	.006622	.006619	.007565	.007572	.4775	.006953	.006933	.007907	.007900
.4530	.006629	.006626	.007572	.007579	.4780	.006960	.006938	.007913	.007906
.4535	.006636	.006633	.007580	.007586	.4785	.006966	.006944	.007920	.007912
.4540	.006643	.006639	.007587	.007593	.4790	.006972	.006950	.007926	.007918
.4545	.006650	.006646	.007594	.007600	.4795	.006978	.006956	.007932	.007924
.4550	.006657	.006653	.007602	.007607	.4800	.006984	.006962	.007938	.007930
.4555	.006664	.006659	.007609	.007614	.4805	.006990	.006967	.007945	.007936
.4560	.006671	.006666	.007616	.007621	.4810	.006996	.006973	.007951	.007942
.4565	.006678	.006672	.007623	.007628	.4815	.007002	.006979	.007957	.007948
.4570	.006685	.006679	.007630	.007635	.4820	.007008	.006985	.007963	.007954
.4575	.006692	.006685	.007638	.007642	.4825	.007014	.006990	.007969	.007960
.4580	.006699	.006692	.007645	.007649	.4830	.007020	.006996	.007975	.007966
.4585	.006706	.006698	.007652	.007656	.4835	.007026	.007002	.007981	.007972
.4590	.006712	.006705	.007659	.007662	.4840	.007032	.007007	.007987	.007978
.4595	.006719	.006711	.007666	.007669	.4845	.007038	.007013	.007994	.007984
.4600	.006726	.006718	.007673	.007676	.4850	.007044	.007018	.008000	.007989
.4605	.006733	.006724	.007680	.007683	.4855	.007050	.007024	.008006	.007995
.4610	.006740	.006731	.007687	.007689	.4860	.007056	.007030	.008012	.008001
.4615	.006746	.006737	.007694	.007696	.4865	.007062	.007035	.008018	.008007
.4620	.006753	.006743	.007701	.007703	.4870	.007068	.007041	.008024	.008012
.4625	.006760	.006750	.007708	.007709	.4875	.007073	.007046	.008029	.008018
.4630	.006767	.006756	.007715	.007716	.4880	.007079	.007052	.008035	.008024
.4635	.006773	.006762	.007722	.007723	.4885	.007085	.007057	.008041	.008030
.4640	.006780	.006769	.007729	.007729	.4890	.007091	.007063	.008047	.008035
.4645	.006787	.006775	.007736	.007736	.4895	.007097	.007068	.008053	.008041
.4650	.006793	.006781	.007743	.007742	.4900	.007102	.007074	.008059	.008047
.4655	.006800	.006788	.007749	.007749	.4905	.007108	.007079	.008065	.008052
.4660	.006807	.006794	.007756	.007756	.4910	.007114	.007085	.008071	.008058
.4665	.006813	.006800	.007763	.007762	.4915	.007120	.007090	.008076	.008063
.4670	.006820	.006806	.007770	.007769	.4920	.007125	.007095	.008082	.008069
.4675	.006826	.006812	.007776	.007775	.4925	.007131	.007101	.008088	.008074
.4680	.006833	.006819	.007783	.007781	.4930	.007137	.007106	.008094	.008080
.4685	.006839	.006825	.007790	.007788	.4935	.007142	.007112	.008099	.008085
.4690	.006846	.006831	.007797	.007794	.4940	.007148	.007117	.008105	.008091
.4695	.006852	.006837	.007803	.007801	.4945	.007154	.007122	.008111	.008096
.4700	.006859	.006843	.007810	.007807	.4950	.007159	.007127	.008116	.008102
.4705	.006865	.006849	.007816	.007813	.4955	.007165	.007133	.008122	.008107
.4710	.006872	.006855	.007823	.007820	.4960	.007170	.007138	.008128	.008113
.4715	.006878	.006861	.007830	.007826	.4965	.007176	.007143	.008133	.008118
.4720	.006884	.006867	.007836	.007832	.4970	.007181	.007149	.008139	.008124
.4725	.006891	.006873	.007843	.007839	.4975	.007187	.007154	.008144	.008129
.4730	.006897	.006879	.007849	.007845	.4980	.007192	.007159	.008150	.008134
.4735	.006903	.006885	.007856	.007851	.4985	.007198	.007164	.008155	.008140
.4740	.006910	.006891	.007862	.007857	.4990	.007203	.007169	.008161	.008145
.4745	.006916	.006897	.007869	.007863	.4995	.007209	.007175	.008166	.008150
.4750	.006922	.006903	.007875	.007870	.5000	.007214	.007180	.008172	.008156

Table A-1 (cont.)

X	STANDARDS	RAWLEY	BOWIE	BEEUKES	X	STANDARDS	RAWLEY	BOWIE	BEEUKES
.5000	.007214	.007180	.008172	.003156	.5250	.007466	.007419	.008424	.008400
.5005	.007220	.007185	.008177	.008161	.5255	.007470	.007423	.008428	.008404
.5010	.007225	.007190	.008183	.008166	.5260	.007475	.007428	.008433	.008409
.5015	.007230	.007195	.008188	.008171	.5265	.007479	.007432	.008437	.008413
.5020	.007236	.007200	.008194	.008177	.5270	.007484	.007436	.008442	.008418
.5025	.007241	.007205	.008199	.008182	.5275	.007489	.007441	.008446	.008422
.5030	.007246	.007210	.008204	.008187	.5280	.007493	.007445	.008451	.008427
.5035	.007252	.007215	.008210	.008192	.5285	.007498	.007450	.008455	.008431
.5040	.007257	.007220	.008215	.008197	.5290	.007502	.007454	.008460	.008435
.5045	.007262	.007225	.008220	.008202	.5295	.007507	.007458	.008464	.008440
.5050	.007268	.007230	.008226	.008208	.5300	.007511	.007462	.008469	.008444
.5055	.007273	.007235	.008231	.008213	.5305	.007516	.007467	.008473	.008448
.5060	.007278	.007240	.008236	.008218	.5310	.007520	.007471	.008478	.008453
.5065	.007283	.007245	.008241	.008223	.5315	.007525	.007475	.008482	.008457
.5070	.007288	.007250	.008247	.008228	.5320	.007529	.007480	.008487	.008461
.5075	.007294	.007255	.008252	.008233	.5325	.007534	.007484	.008491	.008466
.5080	.007299	.007260	.008257	.008238	.5330	.007538	.007488	.008495	.008470
.5085	.007304	.007265	.008262	.008243	.5335	.007543	.007492	.008500	.008474
.5090	.007309	.007270	.008267	.008248	.5340	.007547	.007496	.008504	.008478
.5095	.007314	.007275	.008273	.008253	.5345	.007551	.007501	.008508	.008483
.5100	.007319	.007280	.008278	.008258	.5350	.007556	.007505	.008513	.008487
.5105	.007324	.007284	.008283	.008263	.5355	.007560	.007509	.008517	.008491
.5110	.007330	.007289	.008288	.008268	.5360	.007564	.007513	.008521	.008495
.5115	.007335	.007294	.008293	.008273	.5365	.007569	.007517	.008526	.008499
.5120	.007340	.007299	.008298	.008278	.5370	.007573	.007521	.008530	.008504
.5125	.007345	.007304	.008303	.008283	.5375	.007577	.007525	.008534	.008508
.5130	.007350	.007308	.008308	.008287	.5380	.007582	.007529	.008538	.008512
.5135	.007355	.007313	.008313	.008292	.5385	.007586	.007534	.008542	.008516
.5140	.007360	.007318	.008318	.008297	.5390	.007590	.007538	.008547	.008520
.5145	.007365	.007323	.008323	.008302	.5395	.007594	.007542	.008551	.008524
.5150	.007370	.007327	.008328	.008307	.5400	.007599	.007546	.008555	.008528
.5155	.007375	.007332	.008333	.008312	.5405	.007603	.007550	.008559	.008532
.5160	.007379	.007337	.008338	.008316	.5410	.007607	.007554	.008563	.008536
.5165	.007384	.007342	.008343	.008321	.5415	.007611	.007558	.008567	.008541
.5170	.007389	.007346	.008348	.008326	.5420	.007615	.007562	.008571	.008545
.5175	.007394	.007351	.008352	.008331	.5425	.007619	.007566	.008576	.008549
.5180	.007399	.007355	.008357	.008335	.5430	.007624	.007570	.008580	.008553
.5185	.007404	.007360	.008362	.008340	.5435	.007628	.007574	.008584	.008557
.5190	.007409	.007365	.008367	.008345	.5440	.007632	.007578	.008588	.008560
.5195	.007414	.007369	.008372	.008349	.5445	.007636	.007581	.008592	.008564
.5200	.007418	.007374	.008377	.008354	.5450	.007640	.007585	.008596	.008568
.5205	.007423	.007378	.008381	.008359	.5455	.007644	.007589	.008600	.008572
.5210	.007428	.007383	.008386	.008363	.5460	.007648	.007593	.008604	.008576
.5215	.007433	.007387	.008391	.008368	.5465	.007652	.007597	.008608	.008580
.5220	.007437	.007392	.008396	.008373	.5470	.007656	.007601	.008612	.008584
.5225	.007442	.007396	.008400	.008377	.5475	.007660	.007605	.008616	.008588
.5230	.007447	.007401	.008405	.008382	.5480	.007664	.007609	.008620	.008592
.5235	.007452	.007405	.008410	.008386	.5485	.007668	.007612	.008623	.008596
.5240	.007456	.007410	.008414	.008391	.5490	.007672	.007616	.008627	.008599
.5245	.007461	.007414	.008419	.008395	.5495	.007676	.007620	.008631	.008603
.5250	.007466	.007419	.008424	.008400	.5500	.007680	.007624	.008635	.008607

Table A-1 (cont)

X	STANDARDS	RAWLEY	BOWIE	BEEUKES	X	STANDARDS	RAWLEY	BOWIE	BEEUKES
.5500	.007680	.007624	.008635	.008607	.5750	.007861	.007798	.008812	.008782
.5505	.007684	.007628	.008639	.008611	.5755	.007864	.007801	.008815	.008785
.5510	.007688	.007631	.008643	.008615	.5760	.007868	.007804	.008818	.008788
.5515	.007692	.007635	.008647	.008618	.5765	.007871	.007807	.008821	.008791
.5520	.007696	.007639	.008650	.008622	.5770	.007874	.007810	.008824	.008794
.5525	.007700	.007643	.008654	.008626	.5775	.007877	.007814	.008827	.008798
.5530	.007704	.007646	.008658	.008630	.5780	.007881	.007817	.008831	.008801
.5535	.007707	.007650	.008662	.008633	.5785	.007884	.007820	.008834	.008804
.5540	.007711	.007654	.008666	.008637	.5790	.007887	.007823	.008837	.008807
.5545	.007715	.007657	.008669	.008641	.5795	.007890	.007826	.008840	.008810
.5550	.007719	.007661	.008673	.008645	.5800	.007893	.007829	.008843	.008813
.5555	.007723	.007665	.008677	.008648	.5805	.007897	.007832	.008846	.008816
.5560	.007726	.007668	.008681	.008652	.5810	.007900	.007835	.008849	.008819
.5565	.007730	.007672	.008684	.008655	.5815	.007903	.007838	.008852	.008822
.5570	.007734	.007676	.008688	.008659	.5820	.007906	.007841	.008855	.008825
.5575	.007738	.007679	.008692	.008663	.5825	.007909	.007844	.008858	.008828
.5580	.007741	.007683	.008695	.008666	.5830	.007912	.007847	.008861	.008831
.5585	.007745	.007686	.008699	.008670	.5835	.007915	.007850	.008864	.008834
.5590	.007749	.007690	.008702	.008674	.5840	.007918	.007853	.008867	.008837
.5595	.007753	.007693	.008706	.008677	.5845	.007922	.007856	.008870	.008840
.5600	.007756	.007697	.008710	.008681	.5850	.007925	.007859	.008873	.008843
.5605	.007760	.007700	.008713	.008684	.5855	.007928	.007862	.008876	.008846
.5610	.007764	.007704	.008717	.008688	.5860	.007931	.007865	.008879	.008849
.5615	.007767	.007707	.008720	.008691	.5865	.007934	.007868	.008882	.008852
.5620	.007771	.007711	.008724	.008695	.5870	.007937	.007871	.008885	.008855
.5625	.007775	.007714	.008727	.008698	.5875	.007940	.007874	.008888	.008858
.5630	.007778	.007718	.008731	.008702	.5880	.007943	.007877	.008891	.008861
.5635	.007782	.007721	.008735	.008705	.5885	.007946	.007880	.008894	.008864
.5640	.007785	.007725	.008738	.008709	.5890	.007949	.007883	.008897	.008867
.5645	.007789	.007728	.008741	.008712	.5895	.007952	.007886	.008900	.008870
.5650	.007792	.007732	.008745	.008716	.5900	.007955	.007889	.008902	.008872
.5655	.007796	.007735	.008748	.008719	.5905	.007958	.007891	.008905	.008875
.5660	.007800	.007739	.008752	.008722	.5910	.007961	.007894	.008908	.008878
.5665	.007803	.007742	.008755	.008726	.5915	.007963	.007897	.008911	.008881
.5670	.007807	.007745	.008759	.008729	.5920	.007966	.007900	.008914	.008884
.5675	.007810	.007749	.008762	.008733	.5925	.007969	.007903	.008917	.008887
.5680	.007814	.007752	.008765	.008736	.5930	.007972	.007905	.008919	.008889
.5685	.007817	.007755	.008769	.008739	.5935	.007975	.007908	.008922	.008892
.5690	.007820	.007759	.008772	.008743	.5940	.007978	.007911	.008925	.008895
.5695	.007824	.007762	.008776	.008746	.5945	.007981	.007914	.008928	.008898
.5700	.007827	.007765	.008779	.008749	.5950	.007984	.007917	.008930	.008901
.5705	.007831	.007769	.008782	.008753	.5955	.007986	.007919	.008933	.008903
.5710	.007834	.007772	.008786	.008756	.5960	.007989	.007922	.008936	.008906
.5715	.007838	.007775	.008789	.008759	.5965	.007992	.007925	.008939	.008909
.5720	.007841	.007778	.008792	.008762	.5970	.007995	.007928	.008941	.008911
.5725	.007844	.007782	.008795	.008766	.5975	.007998	.007930	.008944	.008914
.5730	.007848	.007785	.008799	.008769	.5980	.008000	.007933	.008947	.008917
.5735	.007851	.007788	.008802	.008772	.5985	.008003	.007936	.008949	.008920
.5740	.007854	.007791	.008805	.008775	.5990	.008006	.007938	.008952	.008922
.5745	.007858	.007795	.008808	.008779	.5995	.008009	.007941	.008955	.008925
.5750	.007861	.007798	.008812	.008782	.6000	.008011	.007944	.008957	.008928



Table A-1 (cont)

X	STANDARDS	RAWLEY	BOWIE	BEEUWKES	X	STANDARDS	RAWLEY	BOWIE	BEEUWKES
.6000	.08011	.007944	.008957	.008928	.6250	.008135	.008064	.009077	.009048
.6005	.08014	.007946	.008960	.008930	.6255	.008137	.008067	.009079	.009050
.6010	.08017	.007949	.008963	.008933	.6260	.008139	.008069	.009081	.009053
.6015	.08020	.007952	.008965	.008935	.6265	.008141	.008071	.009083	.009055
.6020	.08022	.007954	.008968	.008938	.6270	.008144	.008073	.009085	.009057
.6025	.08025	.007957	.008970	.008941	.6275	.008146	.008075	.009087	.009059
.6030	.08028	.007959	.008973	.008943	.6280	.008148	.008077	.009089	.009061
.6035	.08030	.007962	.008976	.008946	.6285	.008150	.008079	.009091	.009063
.6040	.08033	.007965	.008978	.008948	.6290	.008152	.008082	.009093	.009065
.6045	.08036	.007967	.008981	.008951	.6295	.008154	.008084	.009096	.009067
.6050	.08038	.007970	.008983	.008954	.6300	.008156	.008086	.009098	.009070
.6055	.08041	.007972	.008986	.008956	.6305	.008159	.008088	.009100	.009072
.6060	.08043	.007975	.008988	.008959	.6310	.008161	.008090	.009102	.009074
.6065	.08046	.007977	.008991	.008961	.6315	.008163	.008092	.009104	.009076
.6070	.08049	.007980	.008993	.008964	.6320	.008165	.008094	.009106	.009078
.6075	.08051	.007982	.008996	.008966	.6325	.008167	.008096	.009108	.009080
.6080	.08054	.007985	.008998	.008969	.6330	.008169	.008098	.009110	.009082
.6085	.08056	.007987	.009001	.008971	.6335	.008173	.008100	.009112	.009084
.6090	.08059	.007990	.009003	.008974	.6340	.008175	.008102	.009114	.009086
.6095	.08061	.007992	.009006	.008976	.6345	.008177	.008104	.009116	.009088
.6100	.08064	.007995	.009008	.008979	.6350	.008179	.008106	.009118	.009090
.6105	.08066	.007997	.009010	.008981	.6355	.008181	.008108	.009120	.009092
.6110	.08069	.008000	.009013	.008984	.6360	.008183	.008110	.009122	.009094
.6115	.08071	.008002	.009015	.008986	.6365	.008185	.008112	.009124	.009096
.6120	.08074	.008005	.009018	.008988	.6370	.008187	.008114	.009126	.009098
.6125	.08076	.008007	.009020	.008991	.6375	.008189	.008116	.009128	.009100
.6130	.08079	.008009	.009022	.008993	.6380	.008191	.008118	.009129	.009102
.6135	.08081	.008012	.009025	.008996	.6385	.008193	.008120	.009131	.009104
.6140	.08084	.008014	.009027	.008998	.6390	.008195	.008122	.009133	.009106
.6145	.08086	.008017	.009030	.009000	.6395	.008197	.008124	.009135	.009108
.6150	.08088	.008019	.009032	.009003	.6400	.008199	.008126	.009137	.009110
.6155	.08091	.008021	.009034	.009005	.6405	.008201	.008128	.009139	.009112
.6160	.08093	.008024	.009037	.009007	.6410	.008203	.008130	.009141	.009114
.6165	.08096	.008026	.009039	.009010	.6415	.008205	.008132	.009143	.009116
.6170	.08098	.008028	.009041	.009012	.6420	.008207	.008134	.009144	.009117
.6175	.08100	.008031	.009043	.009014	.6425	.008209	.008135	.009146	.009119
.6180	.08103	.008033	.009046	.009017	.6430	.008211	.008137	.009148	.009121
.6185	.08105	.008035	.009048	.009019	.6435	.008212	.008139	.009150	.009123
.6190	.08107	.008038	.009050	.009021	.6440	.008214	.008141	.009152	.009125
.6195	.08110	.008040	.009052	.009024	.6445	.008216	.008143	.009154	.009127
.6200	.08112	.008042	.009055	.009026	.6450	.008218	.008145	.009155	.009129
.6205	.08114	.008044	.009057	.009028	.6455	.008220	.008147	.009157	.009131
.6210	.08117	.008047	.009059	.009030	.6460	.008222	.008148	.009159	.009132
.6215	.08119	.008049	.009061	.009033	.6465	.008224	.008150	.009161	.009134
.6220	.08121	.008051	.009064	.009035	.6470	.008225	.008152	.009163	.009136
.6225	.08124	.008053	.009066	.009037	.6475	.008227	.008154	.009165	.009138
.6230	.08126	.008056	.009068	.009039	.6480	.008229	.008156	.009166	.009140
.6235	.08128	.008058	.009070	.009042	.6485	.008231	.008157	.009168	.009141
.6240	.08130	.008060	.009072	.009044	.6490	.008233	.008159	.009170	.009143
.6245	.08133	.008062	.009074	.009046	.6495	.008234	.008161	.009171	.009145
.6250	.08135	.008064	.009077	.009048	.6500	.008236	.008163	.009173	.009147

Table A-1 (cont)

X	STANDARDS	RAWLEY	BOWIE	BEEWKES	X	STANDARDS	RAWLEY	BOWIE	BEEWKES
.6500	.008234	.008163	.009173	.009147	.6750	.008313	.008242	.009250	.009226
.6505	.008236	.008164	.009175	.009149	.6755	.008315	.008243	.009252	.009228
.6510	.008238	.008166	.009177	.009150	.6760	.008316	.008244	.009253	.009229
.6515	.008240	.008168	.009178	.009152	.6765	.008318	.008246	.009254	.009231
.6520	.008241	.008170	.009180	.009154	.6770	.008319	.008247	.009256	.009232
.6525	.008243	.008171	.009182	.009156	.6775	.008320	.008248	.009257	.009233
.6530	.008245	.008173	.009183	.009157	.6780	.008322	.008250	.009258	.009235
.6535	.008247	.008175	.009185	.009159	.6785	.008323	.008251	.009260	.009236
.6540	.008248	.008177	.009187	.009161	.6790	.008324	.008252	.009261	.009238
.6545	.008250	.008178	.009188	.009162	.6795	.008326	.008254	.009262	.009239
.6550	.008252	.008180	.009190	.009164	.6800	.008327	.008255	.009264	.009240
.6555	.008253	.008182	.009192	.009166	.6805	.008328	.008256	.009265	.009242
.6560	.008255	.008183	.009193	.009168	.6810	.008330	.008258	.009266	.009243
.6565	.008257	.008185	.009195	.009169	.6815	.008331	.008259	.009268	.009244
.6570	.008258	.008187	.009197	.009171	.6820	.008332	.008260	.009269	.009246
.6575	.008260	.008188	.009198	.009173	.6825	.008333	.008262	.009270	.009247
.6580	.008262	.008190	.009200	.009174	.6830	.008335	.008263	.009271	.009248
.6585	.008263	.008192	.009201	.009176	.6835	.008336	.008264	.009272	.009250
.6590	.008265	.008193	.009203	.009178	.6840	.008338	.008266	.009274	.009251
.6595	.008267	.008195	.009205	.009179	.6845	.008339	.008267	.009275	.009252
.6600	.008268	.008196	.009206	.009181	.6850	.008340	.008268	.009276	.009253
.6605	.008270	.008198	.009208	.009182	.6855	.008341	.008269	.009278	.009255
.6610	.008271	.008200	.009209	.009184	.6860	.008343	.008271	.009279	.009256
.6615	.008273	.008201	.009211	.009186	.6865	.008344	.008272	.009280	.009257
.6620	.008275	.008203	.009212	.009187	.6870	.008345	.008273	.009281	.009259
.6625	.008276	.008204	.009214	.009189	.6875	.008346	.008274	.009283	.009260
.6630	.008278	.008206	.009215	.009190	.6880	.008348	.008276	.009284	.009261
.6635	.008279	.008208	.009217	.009192	.6885	.008349	.008277	.009285	.009262
.6640	.008281	.008209	.009218	.009194	.6890	.008350	.008278	.009286	.009264
.6645	.008283	.008211	.009220	.009195	.6895	.008351	.008279	.009287	.009265
.6650	.008284	.008212	.009222	.009197	.6900	.008353	.008280	.009289	.009266
.6655	.008286	.008214	.009223	.009198	.6905	.008354	.008282	.009290	.009267
.6660	.008287	.008215	.009225	.009200	.6910	.008355	.008283	.009291	.009269
.6665	.008289	.008217	.009226	.009201	.6915	.008356	.008284	.009292	.009270
.6670	.008290	.008218	.009227	.009203	.6920	.008357	.008285	.009293	.009271
.6675	.008292	.008220	.009229	.009204	.6925	.008358	.008286	.009294	.009272
.6680	.008293	.008221	.009230	.009206	.6930	.008360	.008288	.009296	.009273
.6685	.008295	.008223	.009232	.009207	.6935	.008361	.008289	.009297	.009275
.6690	.008296	.008224	.009233	.009209	.6940	.008362	.008290	.009298	.009276
.6695	.008298	.008226	.009235	.009210	.6945	.008363	.008291	.009299	.009277
.6700	.008299	.008227	.009236	.009212	.6950	.008364	.008292	.009300	.009278
.6705	.008301	.008229	.009238	.009213	.6955	.008365	.008293	.009301	.009279
.6710	.008302	.008230	.009239	.009215	.6960	.008367	.008295	.009302	.009280
.6715	.008303	.008232	.009241	.009216	.6965	.008368	.008296	.009303	.009282
.6720	.008305	.008233	.009242	.009218	.6970	.008369	.008297	.009305	.009283
.6725	.008306	.008234	.009243	.009219	.6975	.008370	.008298	.009306	.009284
.6730	.008308	.008236	.009245	.009221	.6980	.008371	.008299	.009307	.009285
.6735	.008309	.008237	.009246	.009222	.6985	.008372	.008300	.009308	.009286
.6740	.008311	.008239	.009248	.009224	.6990	.008373	.008301	.009309	.009287
.6745	.008312	.008240	.009249	.009225	.6995	.008374	.008302	.009310	.009288
.6750	.008313	.008242	.009250	.009226	.7000	.008375	.008303	.009311	.009290

Table A-1 (cont)

X	STANDARDS	RAWLEY	BOWIE	BEEWKES	X	STANDARDS	RAWLEY	BOWIE	BEEWKES
.7000	.008375	.008303	.009311	.009290	.7250	.008423	.008351	.009358	.009339
.7005	.008377	.008304	.009312	.009291	.7255	.008424	.008352	.009359	.009340
.7010	.008378	.008306	.009313	.009292	.7260	.008425	.008353	.009360	.009341
.7015	.008379	.008307	.009314	.009293	.7265	.008426	.008354	.009361	.009342
.7020	.008380	.008308	.009315	.009294	.7270	.008427	.008355	.009362	.009343
.7025	.008381	.008309	.009316	.009295	.7275	.008428	.008356	.009363	.009344
.7030	.008382	.008310	.009317	.009296	.7280	.008429	.008357	.009364	.009345
.7035	.008383	.008311	.009318	.009297	.7285	.008430	.008358	.009365	.009346
.7040	.008384	.008312	.009319	.009298	.7290	.008431	.008359	.009366	.009347
.7045	.008385	.008313	.009320	.009299	.7295	.008432	.008360	.009367	.009348
.7050	.008386	.008314	.009321	.009300	.7300	.008433	.008361	.009368	.009349
.7055	.008387	.008315	.009322	.009301	.7305	.008434	.008362	.009369	.009350
.7060	.008388	.008316	.009323	.009302	.7310	.008435	.008363	.009370	.009351
.7065	.008389	.008317	.009324	.009303	.7315	.008436	.008364	.009371	.009352
.7070	.008390	.008318	.009325	.009304	.7320	.008437	.008365	.009372	.009353
.7075	.008391	.008319	.009326	.009305	.7325	.008438	.008366	.009373	.009354
.7080	.008392	.008320	.009327	.009306	.7330	.008439	.008367	.009374	.009355
.7085	.008393	.008321	.009328	.009307	.7335	.008440	.008368	.009375	.009356
.7090	.008394	.008322	.009329	.009308	.7340	.008441	.008369	.009376	.009357
.7095	.008395	.008323	.009330	.009309	.7345	.008442	.008370	.009377	.009358
.7100	.008396	.008324	.009331	.009310	.7350	.008443	.008371	.009378	.009359
.7105	.008397	.008325	.009332	.009311	.7355	.008444	.008372	.009379	.009360
.7110	.008398	.008326	.009333	.009312	.7360	.008445	.008373	.009380	.009361
.7115	.008399	.008327	.009334	.009313	.7365	.008446	.008374	.009381	.009362
.7120	.008400	.008328	.009335	.009314	.7370	.008447	.008375	.009382	.009363
.7125	.008401	.008329	.009336	.009315	.7375	.008448	.008376	.009383	.009364
.7130	.008402	.008330	.009337	.009316	.7380	.008449	.008377	.009384	.009365
.7135	.008403	.008331	.009338	.009317	.7385	.008450	.008378	.009385	.009366
.7140	.008404	.008332	.009339	.009318	.7390	.008451	.008379	.009386	.009367
.7145	.008405	.008333	.009340	.009319	.7395	.008452	.008380	.009387	.009368
.7150	.008406	.008334	.009341	.009320	.7400	.008453	.008381	.009388	.009369
.7155	.008407	.008335	.009342	.009321	.7405	.008454	.008382	.009389	.009370
.7160	.008408	.008336	.009343	.009322	.7410	.008455	.008383	.009390	.009371
.7165	.008409	.008337	.009344	.009323	.7415	.008456	.008384	.009391	.009372
.7170	.008410	.008338	.009345	.009324	.7420	.008457	.008385	.009392	.009373
.7175	.008411	.008339	.009346	.009325	.7425	.008458	.008386	.009393	.009374
.7180	.008412	.008340	.009347	.009326	.7430	.008459	.008387	.009394	.009375
.7185	.008413	.008341	.009348	.009327	.7435	.008460	.008388	.009395	.009376
.7190	.008414	.008342	.009349	.009328	.7440	.008461	.008389	.009396	.009377
.7195	.008415	.008343	.009350	.009329	.7445	.008462	.008390	.009397	.009378
.7200	.008416	.008344	.009351	.009330	.7450	.008463	.008391	.009398	.009379
.7205	.008417	.008345	.009352	.009331	.7455	.008464	.008392	.009399	.009380
.7210	.008418	.008346	.009353	.009332	.7460	.008465	.008393	.009400	.009381
.7215	.008419	.008347	.009354	.009333	.7465	.008466	.008394	.009401	.009382
.7220	.008420	.008348	.009355	.009334	.7470	.008467	.008395	.009402	.009383
.7225	.008421	.008349	.009356	.009335	.7475	.008468	.008396	.009403	.009384
.7230	.008422	.008350	.009357	.009336	.7480	.008469	.008397	.009404	.009385
.7235	.008423	.008351	.009358	.009337	.7485	.008470	.008398	.009405	.009386
.7240	.008424	.008352	.009359	.009338	.7490	.008471	.008399	.009406	.009387
.7245	.008425	.008353	.009360	.009339	.7495	.008472	.008400	.009407	.009388
.7250	.008426	.008354	.009361	.009340	.7500	.008473	.008401	.009408	.009389

Table A-1 (cont.)

X	STANDARDS	RAWLEY	BOWIE	BEEUKES
.7500	.008460	.008386	.009395	.009376
.7505	.008461	.008387	.009395	.009377
.7510	.008461	.008388	.009396	.009378
.7515	.008462	.008388	.009396	.009378
.7520	.008462	.008389	.009397	.009379
.7525	.008463	.008389	.009398	.009380
.7530	.008464	.008390	.009398	.009380
.7535	.008464	.008391	.009399	.009381
.7540	.008465	.008391	.009399	.009381
.7545	.008465	.008392	.009400	.009382
.7550	.008466	.008392	.009401	.009383
.7555	.008467	.008393	.009401	.009383
.7560	.008467	.008393	.009402	.009384
.7565	.008468	.008394	.009402	.009384
.7570	.008468	.008395	.009403	.009385
.7575	.008469	.008395	.009404	.009386
.7580	.008470	.008396	.009404	.009386
.7585	.008470	.008396	.009405	.009387
.7590	.008471	.008397	.009405	.009387
.7595	.008471	.008397	.009406	.009388
.7600	.008472	.008398	.009406	.009389
.7605	.008472	.008398	.009407	.009389
.7610	.008473	.008399	.009407	.009390
.7615	.008474	.008399	.009408	.009390
.7620	.008474	.008400	.009409	.009391
.7625	.008475	.008400	.009409	.009392
.7630	.008475	.008401	.009410	.009392
.7635	.008476	.008401	.009411	.009393
.7640	.008476	.008402	.009411	.009393
.7645	.008477	.008402	.009411	.009394
.7650	.008477	.008403	.009412	.009395
.7655	.008478	.008403	.009412	.009395
.7660	.008478	.008404	.009413	.009395
.7665	.008479	.008405	.009413	.009396
.7670	.008480	.008405	.009414	.009396
.7675	.008480	.008405	.009414	.009397
.7680	.008481	.008406	.009415	.009397
.7685	.008481	.008406	.009415	.009398
.7690	.008482	.008407	.009416	.009398
.7695	.008482	.008407	.009416	.009399
.7700	.008483	.008408	.009417	.009399
.7705	.008483	.008408	.009417	.009400
.7710	.008484	.008409	.009418	.009400
.7715	.008484	.008409	.009418	.009401
.7720	.008485	.008410	.009419	.009401
.7725	.008485	.008410	.009420	.009402
.7730	.008486	.008410	.009420	.009402
.7735	.008486	.008411	.009420	.009403
.7740	.008487	.008411	.009421	.009403
.7745	.008487	.008412	.009421	.009404
.7750	.008488	.008412	.009422	.009404

X	STANDARDS	RAWLEY	BOWIE	BEEUKES
.7750	.008488	.008412	.009422	.009404
.7755	.008488	.008413	.009422	.009405
.7760	.008488	.008413	.009422	.009405
.7765	.008489	.008413	.009423	.009405
.7770	.008489	.008414	.009423	.009406
.7775	.008490	.008414	.009424	.009406
.7780	.008490	.008415	.009424	.009407
.7785	.008491	.008415	.009425	.009407
.7790	.008491	.008416	.009425	.009408
.7795	.008492	.008416	.009426	.009408
.7800	.008492	.008416	.009426	.009409
.7805	.008493	.008417	.009426	.009409
.7810	.008493	.008417	.009427	.009410
.7815	.008494	.008418	.009427	.009410
.7820	.008494	.008418	.009428	.009411
.7825	.008494	.008418	.009428	.009411
.7830	.008495	.008419	.009429	.009411
.7835	.008495	.008419	.009429	.009412
.7840	.008496	.008420	.009429	.009412
.7845	.008496	.008420	.009430	.009412
.7850	.008497	.008420	.009430	.009413
.7855	.008497	.008421	.009431	.009413
.7860	.008497	.008421	.009431	.009414
.7865	.008498	.008421	.009431	.009414
.7870	.008498	.008422	.009432	.009414
.7875	.008499	.008422	.009432	.009415
.7880	.008499	.008422	.009433	.009415
.7885	.008499	.008423	.009433	.009416
.7890	.008500	.008423	.009433	.009416
.7895	.008500	.008424	.009434	.009416
.7900	.008501	.008424	.009434	.009417
.7905	.008501	.008424	.009435	.009417
.7910	.008501	.008425	.009435	.009418
.7915	.008502	.008425	.009435	.009418
.7920	.008502	.008425	.009436	.009418
.7925	.008503	.008426	.009436	.009419
.7930	.008503	.008426	.009436	.009419
.7935	.008503	.008426	.009437	.009419
.7940	.008504	.008427	.009437	.009420
.7945	.008504	.008427	.009438	.009420
.7950	.008505	.008427	.009438	.009421
.7955	.008505	.008428	.009438	.009421
.7960	.008505	.008428	.009439	.009421
.7965	.008506	.008428	.009439	.009422
.7970	.008506	.008428	.009439	.009422
.7975	.008506	.008429	.009440	.009422
.7980	.008507	.008429	.009440	.009423
.7985	.008507	.008429	.009440	.009423
.7990	.008508	.008430	.009441	.009423
.7995	.008508	.008430	.009441	.009424
.8000	.008508	.008430	.009441	.009424

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<p>Amy Materials and Mechanics Research Center Watertown, Massachusetts 02172 THE ANALYSIS OF CRACK GROWTH DATA WITH APPLICATION TO TRIP STEEL Reinier Beeuwkes, Jr.</p> <p>AD</p> <p>UNCLASSIFIED UNLIMITED DISTRIBUTION</p> <p>Key Words</p> <p>Cracks Crack propagation Materials behavior</p> <p>Material data Models Steel</p> <p>Technical Report AMMRC TR 83-14, March 1983, 57 pp- illus-tables, DA Project IL161102AH42</p> <p>A procedure is advocated to replace the common practice of establishing a rate of crack growth law directly from increments of growth or from slopes of smooth curves drawn through crack length vs number of loading cycles data. In the advocated procedure, a postulated rate equation such as, for example, the power law <math>da/dN \sim (\Delta K)^m</math>, is integrated for the specimen geometry used and values of the integral corresponding to experimental crack lengths are plotted against cycles of loading to reach these lengths. Thus, where, and if, the rate equation is valid the plot will consist of one or more straight line segments for each of which constant parameters may be obtained, or confirmed if theoretical discontinuities unobservable by the usual method. Using Trip Steel data supplied by Syracuse University as an example, it was found that all growth can be proportional to <math>(\Delta K)^2</math> with a proportionality constant that changes discontinuously at various amounts of growth in general conformance to the author's experience and his crack growth law. This conclusion is not invalidated by a gradual decrease in loading during testing, or by the differences in analytical expressions for <math>\Delta K</math> found by different investigators. It is shown that, although a rate proportional to <math>(\Delta K)^4</math> is not that for any segment of the whole growth curve, it represents the envelope to such segments and thus is a sort of overall representation that may be useful in design. The discrepancy between a rate proportional to <math>(\Delta K)^m</math>, <math>m = 3.7</math>, found by Syracuse University and the rate proportional to <math>(\Delta K)^2</math> is explained by showing that the Syracuse method of analysis gives a power corresponding to an envelope.</p>	<p>Amy Materials and Mechanics Research Center Watertown, Massachusetts 02172 THE ANALYSIS OF CRACK GROWTH DATA WITH APPLICATION TO TRIP STEEL Reinier Beeuwkes, Jr.</p> <p>AD</p> <p>UNCLASSIFIED UNLIMITED DISTRIBUTION</p> <p>Key Words</p> <p>Cracks Crack propagation Materials behavior</p> <p>Material data Models Steel</p> <p>Technical Report AMMRC TR 83-14, March 1983, 57 pp- illus-tables, DA Project IL161102AH42</p> <p>A procedure is advocated to replace the common practice of establishing a rate of crack growth law directly from increments of growth or from slopes of smooth curves drawn through crack length vs number of loading cycles data. In the advocated procedure, a postulated rate equation such as, for example, the power law <math>da/dN \sim (\Delta K)^m</math>, is integrated for the specimen geometry used and values of the integral corresponding to experimental crack lengths are plotted against cycles of loading to reach these lengths. Thus, where, and if, the rate equation is valid the plot will consist of one or more straight line segments for each of which constant parameters may be obtained, or confirmed if theoretical discontinuities unobservable by the usual method. Using Trip Steel data supplied by Syracuse University as an example, it was found that all growth can be proportional to <math>(\Delta K)^2</math> with a proportionality constant that changes discontinuously at various amounts of growth in general conformance to the author's experience and his crack growth law. This conclusion is not invalidated by a gradual decrease in loading during testing, or by the differences in analytical expressions for <math>\Delta K</math> found by different investigators. It is shown that, although a rate proportional to <math>(\Delta K)^4</math> is not that for any segment of the whole growth curve, it represents the envelope to such segments and thus is a sort of overall representation that may be useful in design. The discrepancy between a rate proportional to <math>(\Delta K)^m</math>, <math>m = 3.7</math>, found by Syracuse University and the rate proportional to <math>(\Delta K)^2</math> is explained by showing that the Syracuse method of analysis gives a power corresponding to an envelope.</p>
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